Spring 09 Journal

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1 2nd Week

During previous quarter we worked on the formula

$$\sum_1 \sum_2 \sum_3 |\mathbb{L}_{11}|\,|\mathbb{L}_{22}|\,|\mathbb{L}_{33}| \sum_{k_1=0}^{|\alpha_0|} \sum_{\substack{\alpha_1\subset\alpha_0,\\ |\alpha_1|=k_1}} \sum_{k_2=0}^{|\alpha_1|} \sum_{\substack{\alpha\subset\alpha_1,\ \beta\subset\alpha_1,\\ |\alpha|=|\beta|=k_2}} (-1)^{\sigma(\alpha\subset\alpha_1)+\sigma(\beta\subset\alpha_1)}\,|\mathbb{M}_{12}[\alpha_1\setminus\alpha;\alpha_1\setminus\beta]|$$
$$\cdot\,|\mathbb{M}_{23}[\alpha;\beta]|.$$

Let us arrange that formula as follows:

$$\sum_1 \sum_2 |\mathbb{L}_{11}|\,|\mathbb{L}_{22}| \sum_{k_1=0}^{|\alpha_0|} \sum_{\substack{\alpha_1\subset\alpha_0,\\ |\alpha_1|=k_1}} \sum_{k_2=0}^{|\alpha_1|} \sum_{\substack{\alpha\subset\alpha_1,\beta\subset\alpha_1,\\ |\alpha|=|\beta|=k_2}} (-1)^{\sigma(\alpha\subset\alpha_1)+\sigma(\beta\subset\alpha_1)}\,|\mathbb{M}_{12}[\alpha_1\setminus\alpha;\alpha_1\setminus\beta]|$$

$\cdot S_2(\alpha;\beta)$, where $S_2(\alpha;\beta) = \sum_3 |\mathbb{L}_{33}||\mathbb{M}_{23}[\alpha;\beta]|$. By the end of last quarter, an implementation of the factor $S_2(\alpha;\beta) = \sum_3 |\mathbb{L}_{33}||\mathbb{M}_{23}[\alpha;\beta]|$ was successful.

GOAL OF THIS QUARTER

To split the following formula in faster summations.

$$\sum_1 \sum_2 |\mathbb{L}_{11}|\,|\mathbb{L}_{22}| \sum_{k_1=0}^{|\alpha_0|} \sum_{\substack{\alpha_1\subset\alpha_0,\\ |\alpha_1|=k_1}} \sum_{k_2=0}^{|\alpha_1|} \sum_{\substack{\alpha\subset\alpha_1,\ \beta\subset\alpha_1,\\ |\alpha|=|\beta|=k_2}} (-1)^{\sigma(\alpha\subset\alpha_1)+\sigma(\beta\subset\alpha_1)}\,|\mathbb{M}_{12}[\alpha_1\setminus\alpha;\alpha_1\setminus\beta]|$$
$$\cdot S_2(\alpha;\beta).$$

2 3rd Week

We started the splitting of the formula

$$\sum_{1}\sum_{2}|\mathbb{L}_{11}|\,|\mathbb{L}_{22}|\sum_{k_1=0}^{|\alpha_0|}\sum_{\substack{\alpha_1\subset\alpha_0,\\ |\alpha_1|=k_1}}\sum_{k_2=0}^{|\alpha_1|}\sum_{\substack{\alpha\subset\alpha_1,\ \beta\subset\alpha_1,\\ |\alpha|=|\beta|=k_2}}(-1)^{\sigma(\alpha\subset\alpha_1)+\sigma(\beta\subset\alpha_1)}\,|\mathbb{M}_{12}[\alpha_1\setminus\alpha;\alpha_1\setminus\beta]|$$

$\cdot S_2(\alpha;\beta)$, by looking at the signs $(-1)^{\sigma(\alpha\subset\alpha_1)}$, and $(-1)^{\sigma(\beta\subset\alpha_1)}$.
Thus, by using constraints in the structure of $\mathbb{M}_{12}$ we obtain the following splitting $\sigma(\alpha\subset\alpha_1)=\sigma(\alpha^1\subset{\alpha_1}^1)+\sigma(\alpha^2\subset{\alpha_1}^2)+\sigma(\alpha^3\subset{\alpha_1}^3)+|\alpha^2|\,|{\alpha_1}^1|+|\alpha^3|\,(|{\alpha_1}^1|+|{\alpha_1}^2|)$, where $\alpha^i=\alpha\cap G_i$, with $i=1,2,3$.
The idea is to discard from the summations any minor which we know a priori will be zero.

3 4th Week

We have decided to include constraints on the structure of $\mathbb{M}_{23}$ in the current splitting of our main formula. So, we have an additional sum coming from $S_2(\alpha;\beta)$. Also we need to do more computations when combining everything. However, we do not have a definitive formula yet, but we are close.

4 5th Week

Recall that by using the notation $\alpha^i=\alpha\cap G_i$, with $i=1,2,3$ we found that

$$\sigma(\alpha\subset\alpha_1)=\sigma(\alpha^1\subset\alpha_1)+\sigma(\alpha^2\subset\alpha_1)+\sigma(\alpha^3\subset\alpha_1)$$

$$=\sigma(\alpha^1\subset\alpha_1^1)+[\sigma(\alpha^2\subset\alpha_1^2)+|\alpha^2|\,|\alpha_1^1|]+[\sigma(\alpha^3\subset\alpha_1^3)+|\alpha^3|\,|\alpha_1^1|+|\alpha_1^2|]$$

We next apply the constraints $|\alpha^1|=|\beta^1|=0$, and $|\alpha^2|=|\beta^3|$, and $|\alpha^3|=|\beta^2|$; otherwise we get $|\mathbb{M}_{23}[\alpha;\beta]|=0$.

$$\sum_{1}\sum_{2}|\mathbb{L}_{11}|\,|\mathbb{L}_{22}|\sum_{k_1=0}^{|\alpha_0|}\sum_{j_1=0}^{k_1}\sum_{j_2=0}^{k_1-j_1}\sum_{\substack{\alpha_1^1\subset\alpha_0^1,\\ |\alpha_1^1|=j_1}}\sum_{\substack{\alpha_1^2\subset\alpha_0^2,\\ |\alpha_1^2|=j_2}}\sum_{\substack{\alpha_1^3\subset\alpha_0^3,\\ |\alpha_1^3|=k_1-j_1-j_2}}\sum_{m_1=0}^{j_1}\sum_{m_2=0}^{j_2}\sum_{m_3=0}^{k_1-j_1-j_2}$$

$$\sum_{\substack{\alpha^1\subset\alpha_1^1,\\ |\alpha^1|=m_1}}\sum_{\substack{\alpha^2\subset\alpha_1^2,\\ |\alpha^2|=m_2}}\sum_{\substack{\alpha^3\subset\alpha_1^3,\\ |\alpha^3|=m_3}}\sum_{p_1=0}^{j_1}\sum_{p_2=0}^{j_2}\sum_{p_3=0}^{k_1-j_1-j_2}\sum_{\substack{\beta^1\subset\alpha_1^1,\\ |\beta^1|=p_1}}\sum_{\substack{\beta^2\subset\alpha_1^2,\\ |\beta^2|=p_2}}\sum_{\substack{\beta^3\subset\alpha_1^3,\\ |\beta^3|=p_3}}$$

$$\cdot\delta(m_1+m_2+m_3-p_1-p_2-p_3)$$

$$\cdot(-1)^{\sigma(\alpha^1\subset\alpha_1^1)+\sigma(\alpha^2\subset\alpha_1^2)+\sigma(\alpha^3\subset\alpha_1^3)+\sigma(\beta^1\subset\alpha_1^1)+\sigma(\beta^2\subset\alpha_1^2)+\sigma(\beta^3\subset\alpha_1^3)+m_2j_1+m_3(j_1+j_2)+p_2j_1+p_3(j_1+j_2)}$$

$$\cdot\,|\mathbb{M}_{12}[(\alpha_1^1\setminus\alpha^1)\cup(\alpha_1^2\setminus\alpha^2)\cup(\alpha_1^3\setminus\alpha^3);(\alpha_1^1\setminus\beta^1)\cup(\alpha_1^2\setminus\beta^2)\cup(\alpha_1^3\setminus\beta^3)]|$$

$\cdot S_2(\alpha^1\cup\alpha^2\cup\alpha^3;\beta^1\cup\beta^2\cup\beta^3)$, where $\delta(k)$ is 1 if $k=0$, and is 0 otherwise.

This formula looks extremely large; next task is to find, as many as possible, simplifications of it.

5 6th Week

The formula we developed during previous week looks very complicated. Firstly,the splittings brought too many summations. Secondly, we still need to simplify combinations of exponents which lead to an even result which produce positive signs that we must discard. Finally, the constraints in $\mathbb{M}_{12}$, $\mathbb{M}_{23}$, and S_2 yield cancellations on several sums. These tasks not only seem a little bit messy to figure out but also, I think, we do not get a good version to implement yet. Currently we are considering to tackle differently a desirable decomposition.

6 7th Week

Last week we decided to attempt a new strategy, since our previous approach had a lot of simplifications which we might avoid before doing any splitting. Now we need some additional notation. Let $\gamma^2=\alpha_1^2\setminus\alpha$, $\delta^2=\alpha_1^2\setminus\beta$, and recall that $N_i=|G_i|$. Since $\alpha_1\setminus\alpha\subset G_1\cup G_2$, $\alpha_1\setminus\beta\subset G_1\cup G_2$, and $\alpha,\beta\subset G_2\cup G_3$, we have that $|\alpha^2|=|\beta^2|=|\alpha_1^3|$, and $|\alpha_1^1|=|\gamma^2|=|\delta^2|$.

Now instead of expressing the minors in terms of complements, we compute them with respect to disjoint unions. So our current formula looks like:

$$\sum_{|\alpha_1^1|=0}^{N_1} \sum_{\alpha_1^1 \subset G_1} \sum_{|\alpha_1^3|=0}^{N_3} \sum_{\alpha_1^3 \subset G_3} \sum_{\substack{\gamma^2 \subset G_2, \\ |\gamma^2|=|\alpha_1^1|}} \sum_{\substack{\delta^2 \subset G_2, \\ |\delta^2|=|\alpha_1^1|}} \sum_{\substack{|\alpha^2|=|\alpha_1^3|, \\ \alpha^2 \cap \gamma^2 = \emptyset}} \sum_{\substack{|\beta^2|=|\alpha_1^3|, \\ \beta^2 \cap \delta^2 = \emptyset}} sgn(?)$$

$\cdot\, |\mathbb{M}_{12}[\alpha_1^1 \cup \gamma^2; \alpha_1^1 \cup \delta^2]|\, |\mathbb{M}_{23}[\alpha^2 \cup \alpha_1^3; \beta^2 \cup \alpha_1^3]|$, where the sign of the terms is to be determined.

7 8th Week

This time we include the missing signs in our last formula:

$$\sum_{|\alpha_1^1|=0}^{N_1} \sum_{\alpha_1^1 \subset G_1} \sum_{|\alpha_1^3|=0}^{N_3} \sum_{\alpha_1^3 \subset G_3} \sum_{\substack{\gamma^2 \subset G_2, \\ |\gamma^2|=|\alpha_1^1|}} \sum_{\substack{\delta^2 \subset G_2, \\ |\delta^2|=|\alpha_1^1|}} \sum_{\substack{|\alpha^2|=|\alpha_1^3|, \\ \alpha^2 \cap \gamma^2 = \emptyset}} \sum_{\substack{|\beta^2|=|\alpha_1^3|, \\ \beta^2 \cap \delta^2 = \emptyset}} (-1)^{\sigma(\alpha^2 \subset \alpha^2 \cup \gamma^2) + \sigma(\beta^2 \subset \beta^2 \cup \delta^2)}$$

$\cdot\, |\mathbb{M}_{12}[\alpha_1^1 \cup \gamma^2; \alpha_1^1 \cup \delta^2]|\, |\mathbb{M}_{23}[\alpha^2 \cup \alpha_1^3; \beta^2 \cup \alpha_1^3]|$

8 9th Week

I am now summarizing what I did during the quarter in order to submit our final report and my final journal. However, I am still trying to figure out additional simplifications from our new splitting.

9 9th Week

As a conclusion, we cannot say that we completed our goal this quarter. We tried several splittings which might be now just two versions. The theoretical work was sometimes hard and frustrating, however we now know better the obstacles. Also an implementation of the first version has failed, and we have to figure out and correct every errors the code has. Moreover, we found that a lot (or perhaps all) of the code's outcomes are erratically whole numbers...!?, so the code certainly has a mistake.

Although, the latest version of the formula looks promising, we still need to implement it. That should be a good task for Fall quarter, I guess.