Fall 2008 Journal

Benigno Parra

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1 2nd Week

Our current goal is to figure the cost to computing, when $\mathbb{A}$, and $\mathbb{B}$ are of low ranks, the value of the determinant,

$$|\mathbb{I}+\mathbb{A}+\mathbb{B}| = \sum_{k_1=0}^{|\alpha_0|} \sum_{\substack{\alpha_1\subset\alpha_0,\\ |\alpha_1|=k_1}} \sum_{k_2=0}^{|\alpha_1|} \sum_{\substack{\alpha\subset\alpha_1,\ \beta\subset\alpha_1,\\ |\alpha|=|\beta|=k_2}} (-1)^{\sigma(\alpha\subset\alpha_1)+\sigma(\beta\subset\alpha_1)} \, |\mathbb{A}[\alpha_1\setminus\alpha;\alpha_1\setminus\beta]|\cdot|\mathbb{B}[\alpha;\beta]| \tag{1}$$

2 3rd Week

I have analyzed the complexity of previous week formula when ranks of $\mathbb{A}$ and $\mathbb{B}$ are one. We divided the cases depending on k_1 and k_2 values. By summarizing we might say that when $k_1.k_2 = 0$ the complexity is at most n. I am still trying to fully understand the case when $k_1 = 2$ and $k_2 = 1$.

3 4th Week

Regarding the computation of the complexity when ranks of $\mathbb{A}$ and $\mathbb{B}$ are one, we have completed several cases and fixed some details on previous week calculation. We are about finishing the calculations from the case where $k_1 = 2$ and $k_2 = 1$.

4 5th Week

We finally computed all of the cases, and realized that we found some expressions which went back to previous semester computations. In addition, we might say that

since the case when ranks of $\mathbb{A}$ and $\mathbb{B}$ are one is already getting very complicated expressions, and missing a lot of higher rank cases, we possibly need to make some adjustements on our current task.

5 6th Week

We have decided to change our approach since previous one was leading to more complicated expressions. Now we could make certain assumptions in the (22)-nd formula from Dr Mohlenkamp's draft (Sept. 4, 2008), i.e.

$$\sum_1\sum_2\sum_3 |\mathbb{L}_{11}||\mathbb{L}_{22}||\mathbb{L}_{33}| \begin{vmatrix} \mathbb{I} & {\mathbb{L}^{-1}}_{11}\mathbb{L}_{12} & 0 \\ {\mathbb{L}^{-1}}_{22}\mathbb{L}_{21} & \mathbb{I} & {\mathbb{L}^{-1}}_{22}\mathbb{L}_{23} \\ 0 & {\mathbb{L}^{-1}}_{33}\mathbb{L}_{32} & \mathbb{I} \end{vmatrix}.$$ These assumptions are about ranks and by using physical facts involved in the current application. We might use certain unitary matrices for simplifying some block-minors from the previous bigger determinant, and figuring those matrices out is our task for the following days.

6 8th Week

I am currently working with Householder's transformations. This method will help to convert some blocks in the most right determinant from the formula:

$$\sum_1\sum_2\sum_3 |\mathbb{L}_{11}||\mathbb{L}_{22}||\mathbb{L}_{33}| \begin{vmatrix} \mathbb{I} & {\mathbb{L}^{-1}}_{11}\mathbb{L}_{12} & 0 \\ {\mathbb{L}^{-1}}_{22}\mathbb{L}_{21} & \mathbb{I} & {\mathbb{L}^{-1}}_{22}\mathbb{L}_{23} \\ 0 & {\mathbb{L}^{-1}}_{33}\mathbb{L}_{32} & \mathbb{I} \end{vmatrix}.$$

In fact we use that strategy in order to get in blocks like ${\mathbb{L}^{-1}}_{22}\mathbb{L}_{23}$ a $k \times k$ minor just in one of the corners sub-blocks of it, and zeros elsewhere, when k is the rank of $\mathbb{L}_{23}$. That method is precisely working in finding some nice unitary matrices which allow us to multiply on the right and on the left, and since they have determinant one the strategy perfectly works. We have not finished all the computations, but soon will be done.

7 9th Week

Since one of the goals of this project is to implement codes and to decrease the complexity of every computation, my current task is to figure how we will apply Householder's transformations:

Let $\mathbb{A} = [a_{ij}]$ be a non-full rank $n \times n$ matrix, and denote by $\mathbb{A}_1$ its first column. Let β be $-\frac{a_{11}}{|a_{11}|}||\mathbb{A}_1||$, and consider the vector $v = \frac{\sqrt{2}}{||\mathbb{A}_1 - \beta e_1||}\mathbb{A}_1 - \beta e_1$, where e_1 is the unitary vector whose first entry is one and zeros elsewhere. Then the unitary matrix u_1 defined by $\mathbb{I} - vv^*$ is what we repeatedly use for reducing some of $\mathbb{A}$, where v^* is the conjugate transpose of v.

Thus, $u_1\mathbb{A} = \begin{bmatrix} a'_{11} & \\ 0 & \mathbb{A}' \\ \vdots & \\ 0 & \end{bmatrix}$.

By repeating this process to each remaining block-matrix on the right side, we finally get

$$u_n \dots u_2 u_1 \mathbb{A} = \begin{bmatrix} a'_{11} & a'_{12} & \dots & a'_{1n} \\ 0 & a'_{22} & \dots & a'_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & a'_{nn} \end{bmatrix}. \tag{2}$$

Notice that since $\mathbb{A}$ is not full rank, there exits k such that $a'_{ii} = 0$ for $i = k, \dots, n$.

Finally, $\mathbb{U} = u_n \dots u_2 u_1$ is the desired unitary matrix. Previous process carries some details that should be adjusted. Firstly the determinant of a unitary matrix might be -1 instead of 1. Secondly, when applying Householder's transformations it is possible to find a zero entry on the diagonal but there might be non-zero elements below that position, in such a case we have to make a permutation on rows which leads to a negative sign that we should be aware of. On making the reduction on the right side columns we might use transposition of matrices in order to get the unitary matrix that we have to multiply on the right side of $\mathbb{UA}$.