1 2nd Week

We are working on the alternate formula for computing $Det(A + B)$, instead of using the one Dr Mohlenkamp and assistants were deriving in the previous quarter. We have already checked how the formula $Det(I + A + B)$ works for the $3 \times 3$ matrices case. Possibly we need to work a few more examples in order to get a precise formula in the case of $Det(A + B + C)$.

2 3rd Week

After checking the formula

$$det(A + B) = \sum_{r=0}^{n} \sum_{\alpha, \beta} (-1)^{s(\alpha) + s(\beta)} det(A[\alpha, \beta]) det(B(\alpha, \beta))$$

for some small size matrices, we moved to consider the addition of three matrices and how the previous formula looks like in this new setting. By using a suggestion from Dr Mohlenkamp, we got the following equalities, $det(A + B + C) =$

$$\sum_{r=0}^{n} \sum_{\alpha, \beta} (-1)^{s(\alpha) + s(\beta)} det(A[\alpha, \beta]) \{ \sum_{t=0}^{r} (-1)^{s(\gamma) + s(\delta)} det(B[\gamma, \delta]) det(C(\gamma, \delta)) \}$$

$$= \sum_{r=0}^{n} \sum_{\alpha, \beta} \sum_{t=0}^{r} (-1)^{s(\alpha) + s(\beta) + s(\gamma) + s(\delta)} det(A[\alpha, \beta]) det(B[\gamma, \delta]) det(C(\gamma, \delta)), \quad (2)$$

where $\gamma$, and $\delta$ are strictly increasing sequences contained in the complement of the sequences $\alpha$, and $\beta$ respectively, and $s | \gamma |$ is the sum of the actual positions (related to the new complementary submatrix) of the rows determined by $\gamma$, likewise for $s | \delta |$. 

1
3 4th Week

We are improving the notation of the formula and also adjusting the way the formula should be displayed by \( LaTex \). The formula for the case of three matrices looks as follows. Let \( \Lambda_n \) be the ordered set \( \{1, \ldots, n\} \), and consider for each subset \( \alpha \subset \Lambda_n \), same increasing order. Let \(|\alpha|\) be the cardinality of \( \alpha \), and \( \sigma(\alpha) = \sum \alpha_i \). Then

\[
det(A + B + C) = \sum_{k_1=0}^{n} \sum_{k_2=0}^{k_1} (-1)^{|\alpha_1|+|\beta_1|+|\alpha_2|+|\beta_2|} \left| A[\Lambda_n \setminus \alpha_1; \Lambda_n \setminus \beta_1] \right| \times \left| B[\alpha_1 \setminus \beta_1; \beta_1 \setminus \beta_2] \right| \left| C[\alpha_2 \setminus \beta_2] \right|, \tag{3}
\]

where \( |\alpha_2| \) is precisely adding the actual position of the rows from \( \alpha_2 \) in the new submatrix.

4 5th Week

The latest version of the formula for the case of three matrices looks as follows. Let \( \Lambda_n \) be the ordered set \( \{1, \ldots, n\} \), and consider for each subset \( \alpha \subset \Lambda_n \), same increasing order. Let \(|\alpha|\) be the cardinality of \( \alpha \), and \( \sigma(\alpha; \beta) = \sum (\alpha_i + \beta_i) \). Then

\[
det(A + B + C) = \sum_{k_1=0}^{n} \sum_{k_2=0}^{k_1} (-1)^{|\alpha_1; \beta_1|} \left| A[\Lambda_n \setminus \alpha_1; \Lambda_n \setminus \beta_1] \right| \times \sum_{k_2=0}^{k_1} \sum_{k_2=0}^{k_1} (-1)^{|\alpha_2; \beta_2|} \left| B[\alpha_1 \setminus \beta_1; \beta_1 \setminus \beta_2] \right| \left| C[\alpha_2 \setminus \beta_2] \right|, \tag{4}
\]

where \( |\alpha_2; \beta_2| \) stands for the summation of all the actual positions of the rows from \( \alpha_2 \), and columns from \( \beta_2 \) in the new submatrix. Also \( det(A + B + C) = \)

\[
\sum_{k_1=0}^{n} \sum_{k_2=0}^{k_1} \sum_{k_2=0}^{k_1} (-1)^{|\alpha_1; \beta_1|+|\alpha_2; \beta_2|} \left| A[\Lambda_n \setminus \alpha_1; \Lambda_n \setminus \beta_1] \right| \times \left| B[\alpha_1 \setminus \beta_1; \beta_1 \setminus \beta_2] \right| \left| C[\alpha_2 \setminus \beta_2] \right|, \tag{5}
\]

where \( |\alpha_2; \beta_2| \) stands for the summation of all the actual positions of the rows from \( \alpha_2 \), and columns from \( \beta_2 \) in the new submatrix. Since we are interested in the case when the identity matrix is one of the matrices, we particularly get that
\[ \det(I + A) = \sum_{k=0}^{n} \sum_{|\alpha| = k, \alpha \subseteq \Lambda_n} |A[\alpha; \alpha]|, \quad (6) \]

where if \( k = 0 \) then (there is no lines to choose from \( A \)) the summand is \( 1 = |I| \).

Another special case is when we consider the matrix

\[
L = \begin{bmatrix}
I & L_{12} & 0 \\
L_{21} & I & L_{23} \\
0 & L_{32} & I
\end{bmatrix}, \quad (7)
\]

where \( I \) is the \( k \times k \) matrix. Then the previous formula leads

\[ \det(I + L) = \sum_{k=0}^{n} \sum_{|\alpha| = k, \alpha \subseteq \Lambda_n} |L_{12}[\alpha; \alpha]| + |L_{21}[\alpha; \alpha]| + |L_{23}[\alpha; \alpha]| + |L_{32}[\alpha; \alpha]|, \quad (8) \]

where if \( k = 0 \) then the summand is \( 1 = |I| \).

5 6th Week

We are still working on getting a better expression for the sign in \((-1)^{\sigma_{|\alpha_2|} + \sigma_{|\beta_2|}}\) or

or relate it to the other sign \((-1)^{\sigma(\alpha_1; \beta_1)}\) in the following formula, where now \( \alpha_0 \) is

the ordered set \( \{1, \ldots, n\} \). Then

\[
\det(A + B + C) = \sum_{k_1=0}^{n} \sum_{\alpha_1 \subseteq \alpha_0, \beta_1 \subseteq \alpha_0, |\alpha_1| = |\beta_1| = k_1} (-1)^{\sigma(\alpha_1) + \sigma(\beta_1)} |A[\alpha_0 \setminus \alpha_1; \alpha_0 \setminus \beta_1]| \times \\
\sum_{k_2=0}^{k_1} \sum_{\alpha_2 \subseteq \alpha_1, \beta_2 \subseteq \beta_1, |\alpha_2| = |\beta_2| = k_2} (-1)^{\sigma_{|\alpha_2|} + \sigma_{|\beta_2|}} |B[\alpha_1 \setminus \alpha_2; \beta_1 \setminus \beta_2]| |C[\alpha_2; \beta_2]|, \quad (9)
\]

where \( \sigma_{|\alpha_2|} + \sigma_{|\beta_2|} \) stands for the summation of all the actual positions of the rows

from \( \alpha_2 \), and columns from \( \beta_2 \) in the new sub-matrix. We also worked with the case

of the matrix

\[
L = \begin{bmatrix}
I & L_{12} & 0 \\
L_{21} & I & L_{23} \\
0 & L_{32} & I
\end{bmatrix}, \quad (10)
\]

where \( I \) is the \( t \times t \) identity matrix. Then the previous formula leads
\[
det(I + L) = \sum_{k=0}^{t} \sum_{\substack{|\alpha| = k \\alpha \subseteq \alpha_0}} |L_{12}[\alpha; \alpha]| + |L_{21}[\alpha; \alpha]| + |L_{23}[\alpha; \alpha]| + |L_{32}[\alpha; \alpha]|, \tag{11}
\]
where if \( k = 0 \) then the summand is \( 1 = |I| \).

6 7th Week

We earlier got the formula

\[
det(I + A) = \sum_{k=0}^{n} \sum_{\substack{|\alpha| = k \\alpha \subseteq \alpha_0}} |A[\alpha; \alpha]|, \tag{12}
\]

where if \( k = 0 \) then (there is no lines to choose from \( A \)) the summand is \( 1 = |I| \).

Now we have tried to use it for the following case: consider the matrix

\[
L = \begin{bmatrix}
I & L_{12} & 0 \\
L_{21} & I & L_{23} \\
0 & L_{32} & I
\end{bmatrix}, \tag{13}
\]

where \( I \) is the \( t \times t \) identity matrix. A nice formula for this case has not been easily for me, and sometimes I have mistaken a good formula. In particular, the formulas in the previous week are incorrect. Lately I have been using \( A = L - I \) into my first formula, but my calculations are not complete yet. I hope in today’s discussion we get a more clear picture about.

7 8th Week

I have been working on a formula for \( |I + L| \), where

\[
L = \begin{bmatrix}
I & L_{12} & 0 \\
L_{21} & I & L_{23} \\
0 & L_{32} & I
\end{bmatrix}. \tag{14}
\]

Some experiments suggest that \( |I + L| = 1 \pm |L_{12}| |L_{21}| \pm |L_{23}| |L_{32}| \), where the signs depends on the relabeled rows and columns from \( L_{ij} \) as before, but I need to complete my reasoning. We are also working on our poster. I hope the coming week we find a nice formula for \( |I + L| \).
8 9th Week

I have been working on a formula for $|I + L|$, where

$$L = \begin{bmatrix} I & L_{12} & 0 \\ L_{21} & I & L_{23} \\ 0 & L_{32} & I \end{bmatrix}.$$  \hspace{1cm} (15)

We are still experimenting, and I think we are not close to reach a nice formula. In order to get at least some partial results, our current task is to figure which are those non-zero minors that we can build up from $L$. Our conjecture is that we may get non-zeros minors whose orders are either $2t$ or running from 1 to $t$, where $t$ is the dimension of the square matrices $L_{ij}$.

Another task that we did by the end of this quarter was a poster session in the Department of Mathematics where we shared the results of our particular research. I had never done a poster, so it was a great experience. Firstly Chelsie (mi partner in this project during this quarter) and I had to switch the results from our journals into a poster style. We had some troubles with the final printing of the poster, possibly due to a lack of enough preparation time and that I had to do a second poster. But beside that the session went well, we got a few questions from the public however the interaction with the people was wonderful.