Journal-Spring 2007

Kristen M. Fox

June 4, 2007

1 Week 1: March 27-April 2, 2007-Introduction to Research Goals and LaTeX

I tried to make small article documents to become familiar with the program LaTeX . I used LaTeX to write a mathematical autobiography and this weeks journal entry.

Concerning our research goals, I read for understanding "Trigonometric Identities and Sums of Separable Functions."

My group met and started working on geometrically proving Lemma 1:

$$\sin(A+B) = \frac{\sin(A)\sin(B+\beta-\alpha)}{\sin(\beta-\alpha)} + \frac{\sin(A+\alpha-\beta)\sin(B)}{\sin(\alpha-\beta)} \tag{1}$$

For this I tried to construct a geometric proof similar to Professor Schaumberger's 1979 proof found in "Geometric Proofs of the Formulas for $\sin(x+y)$and $\cos(x+y)$" using the following equality to construct lengths of the sides of the triangles:

$$\cos(x) = \frac{\sin(B+\beta-\alpha)}{\sin(\beta-\alpha)} \tag{2}$$

One triangle has angles measuring A, $\beta-\alpha$ and 90. The other triangle has angles measuring B, $\alpha-\beta$, and 90. The two right angles are placed back to back with the two triangles having the same height. Using algebra and substitution, we got something that is similar to the proof but a couple of variables are switched, meaning that there is more work to do. I wonder if this method can be used though because it limits the values for the measures of the angles in the triangle.

2 Week 2: April 3-10, 2007-Generalized Two Variable Proof

During the weekly meeting, we discovered that the geometric proof that we did last week was not a general proof, but rather only applicable when $\gamma = \pi/2$.

We were asked to think about the geometric representation from the past week and add a line from a vertex not containing A or B to the opposite side of the triangle. The angle created by this line with the line of the height of the large triangle is γ.

On Friday we worked on our proof using the angle measurements I found in terms of A, B and γ and the geometric representation we constructed Tuesday during our meeting. We calculated side lengths in terms of the angle measurements and used them to represent the sum of the area of the two triangles. From this we obtained a really long equation. We tried to simplify the expression by combining like terms and using trigonometric identities and properties. We still obtained a complex formula that does not look like the identity, Lemma 1.

This week I wrote out my Numerical Linear Analysis homework using LaTeX to get a better understanding of how to use mathematical notation in the program. I also downloaded TexnicCenter, containing a version of LaTeX which I prefer to PCTex.

3 Week 3: April 11-17, 2007-Begin Proof of Three Variables

I tried to solve the proof of the geometric identity for

$$\begin{aligned}\sin(x+y+z) = {} & \sin(x)\frac{\sin(y+\beta-\alpha)}{sin(\beta-\alpha)}\frac{\sin(z+\gamma-\alpha)}{\sin(\gamma-\alpha)} \\ & + \frac{\sin(x+\alpha-\beta)}{\sin(\alpha-\beta)}\sin(y)\frac{\sin(z+\gamma-\beta)}{\sin(\gamma-\beta)} \\ & + \frac{\sin(x+\alpha-\gamma)}{\sin(\alpha-\gamma)}\frac{\sin(y+\beta-\gamma)}{sin(\beta-\gamma)}sin(z).\end{aligned} \tag{3}$$

We used a large triangle with one angle divided into x, y and z with the trisecting lines meeting the base at angles θ and ϕ. All interior angles are calculated based on x, y, z, θ and ϕ. Similar to the double angle identity proof, the law of sines was used to find the length of the sides of the triangle. In this case, θ and ϕ are dependent on each other, $\theta = \phi + y$. A couple of fractions reduced too much in this case.

As a group, we tried to geometrically prove the three angle identity by making the free variables independent of each other. With this approach, we still have a triangle with one angle divided into x, y and z, but the base of the triangle is not a straight line. Again we determined the measure of all interior angles based on the variables and used the law of sines to find lengths in the triangle. After reduction, it appears as if we have half the product in the identity present.

4 Week 4: April 18-24, 2007-Trying to Construct a Geometric Picture for Three Variables

This week I read the induction proof for the multivariable theorem to try and better understand the relationships between different terms in the three angle identity proof.

We tried to geometrically prove the three angle identity by labeling angles in our geometry to see relationships between them and try to figure out how to represent angles that are needed in the proof in the geometry. We are still having difficulties labeling $x + \theta+\phi$, $y + \theta+\phi$ and $z + \theta+\phi$. We found out how to geometrically show one at a time, such as $y + \theta+\phi$, but not all three in the same geometric figure.

5 Week 5: April 25-May 1, 2007-Necessity of Two Variables Found within One Angle

During our group meeting we went through the induction proof of the sin of n summed variables with $n - 1$ free parameters. Later, as a group we tried to connect parts of the induction proof to a geometric picture to obtain a geometric proof.

I translated the proof into statements where $n = 3$ as opposed to its original general form, n. With this I tried to picture how the geometry needs to appear with n variables. I added what I think the $n - 1$ free parameters would look like in the geometry. In this geometry, the only angle I am missing is $x + \theta + \phi$. Before we thought it was bad when a combination of 2 variables and one or more free parameters resulted as an angle. Now I think that it is necessary to have this occur because of the induction proof. I wonder if this angle is necessary or it can be derived by translating the $n - 1$ case to the geometry from the induction proof.

6 Week 6: May 2-8,2007-Discovery of a Picture That Has All Needed Angles

As a group we formulated a picture that has all of the angles required in the identity by placing angle z next to x and y. Other group members think that this model can be used when more variables are added. I do not think that this is a generalized model because each added variable would need to be next to every other previous variable. I think that you get two variables together in the general case by using the $n - 1$ free parameters to form triangles that include each combination of two variables.

7 Week 7: May 9-15, 2007-Application of Induction Step by Recovering Missing Angles

As a group we determined that it is not necessary to have a triangle that has all of the angles present because of lemma 1 we proved geometrically earlier this quarter. Any combination of more than one variable can be divided into its constituent parts using lemma 1.

Since I formulated my model from two weeks ago using inductive thinking, I used this model to try and substitute different angle and length equalities using the law of sines. I know the answer is lurking right around the corner. There is some trick hiding or some equality that is alluding me that will make it all work. It just needs to be uncovered.

8 Week 8: May 16-22, 2007-Using Four Variables and Determining Pictures

Chelsie came up with a way to show that three variables can be transformed into the identity by using lemma 1 three times and canceling out like terms. We applied this concept in our group meeting Friday, Chelsie and I working on the board and Cody working on paper, eventually passing us. While Cody continued to work on the four variables, Chelsie and I tried to formulate triangles that illustrate these transformations. Chelsie has a good concept of inserting a line with a needed angle but I do not think that it can be used how it currently is because it does not share any lengths with any other triangles, a condition needed when using the law of sines. Maybe the line can be shifted to an intersection of lines so that it shares a side with another triangle. I continued to try to find a illustration during the weekend to no avail.

9 Week 9: May 23-29, 2007-Found all Four Variable Terms and More

On Friday we came up with all of the terms needed for the four variable identity. The only problem is that we have four terms left over that we do not know how they cancel out. I came up with a picture that has all needed angles and splits that was necessary for the three variable proof and put it into GSP.

10 Week 10: May 30-June 3, 2007-Finishing up

We each presented a part of our research work to the other research groups Wednesday. I presented the geometric proof of the sine of the sum of two angles. I created power point slides for the presentation and e-mailed them to Cody. He did not use the version of the powerpoint that I e-mailed him with my slides. Instead, we used a version of the powerpoint which had a copy of the GSP proof he presented earlier in the quarter. I had removed some of the steps that were in the powerpoint we used for the presentation so I talked more than was necessary during the presentation. On the positive side, our division of the powerpoint did not detract from its presentation.