# Journal of Cody Chappell for Spring 2007

## 1 Week of March 26, 2007

### 1.1 Introductions, Trigonometric Identities and Learning LaTeX

I met with Dr. Mohlenkamp and the other two undergraduate students participating in the project. We were given our goals to complete by the end of this quarter. I read over "Trigonometric Identities and Sums of Separable Functions"[1] and a handout presenting the geometric proofs for $\sin(x+y)$ and $\cos(x+y)$. The entire research group met and we shared brief introductions with each other.

We three undergraduate students focused on geometrically proving the identity

$$s(A+B) = \frac{\mathrm{s(A)s(B}+\beta-\alpha)}{\mathrm{s}(\beta-\alpha)} + \frac{\mathrm{s(A}+\alpha-\beta)\mathrm{s(B)}}{\mathrm{s}(\alpha-\beta)} \tag{1}$$

where $s(x) = \sin(x)$. We worked toward a proof but are not quite completely finished.

I met with Jyothsna to learn the basics of LaTeX. She taught me enough so that I could begin using LaTeX. I then was able to write my mathematical autobiography.

## 2 Week of April 2, 2007

### 2.1 More with Trigonometric Identities

We three undergraduate students met with Dr. Mohlenkamp and discussed are work on proving equation (1) geometrically. Our work was not quite accurate because we had $\alpha-\beta$ and $\beta-\alpha$ dependant upon A and B. We discussed another starting point from which to formulate the proof.

It is with great difficulty, however, that we attempt to prove this identity. In creating a geometric picture and using trigonometric functions, we are creating an intricate equation which is difficult to compute. It appears that there must be a less complicated equation than the one with which we are currently working.

I received the book Proof Without Words: Exercises in Visual Thinking [2] and have reviewed the proofs of $\sin(x+y)$. In further discussion with the other group members, this new information will hopefully prove useful in uncovering the geometric proof of formula (1).

## 3 Week of April 9, 2007

### 3.1 Solution Found

We were finally able to prove formula (1) geometrically. The key was to use the Law of Sines to determine the lengths. The formula for the Law of Sines states

$$\frac{\sin(\mathrm{A})}{\mathrm{a}} = \frac{\sin(\mathrm{B})}{\mathrm{b}} = \frac{\sin(\mathrm{C})}{\mathrm{c}}. \tag{2}$$

From this, we could use the Law of Sines formula to find $\sin(A+B)$. We greatly rejoiced at discovering the geometric proof.

On Wednesday, Chelsie and I attempted to explain to the other members of the group about what we were working on. I feel we did an adequate job explaining our discovery of the geometric proof of equation (1), but we failed to explain the underlying reason for our search for this proof. We were assigned to discover this reasoning.

Later in the week, we three undergraduate students began trying to geometrically prove the identity

$$\begin{aligned}\sin(x+y+z) = {} & \sin(x)\frac{\sin(\mathrm{y}+\beta-\alpha)}{\sin(\beta-\alpha)}\frac{\sin(\mathrm{z}+\gamma-\alpha)}{\sin(\gamma-\alpha)} \\ & + \frac{\sin(\mathrm{x}+\alpha-\beta)}{\sin(\alpha-\beta)}\sin(y)\frac{\sin(\mathrm{z}+\gamma-\beta)}{\sin(\gamma-\beta)} \\ & + \frac{\sin(\mathrm{x}+\alpha-\gamma)}{\sin(\alpha-\gamma)}\frac{\sin(\mathrm{y}+\beta-\gamma)}{\sin(\beta-\gamma)}\sin(z).\end{aligned} \tag{3}$$

We have investigated possible geometric drawings but have yet to discover the proof.

# 4 Week of April 16, 2007

## 4.1 Trigonometric Identity with Three Variables

We continue to struggle with the proof of equation (3). It is difficult to branch from the proof of the trigonometric identity with two variables to one with three variables. It appears that this current identity should have a geometric proof very similar to the previous equation (1). We have attempted to create a variety of figures originating from the previous geometric proof. However when attempting to compute, the use of the Law of Sines has complicated the proof.

# 5 Week of April 23, 2007

## 5.1 More with the Trigonometric Identity with Three Variables

We met with Dr. Mohlenkamp on Tuesday and muddled through the proof by induction of the $\sin(A+B)$. This proof shows that the sine of the sum of any number $n$ angles has a similar identity. We three undergraduate students met on Friday to discuss this proof but in terms of three angles — $A$, $B$, and $C$. I feel that we made some positive steps toward further understanding this proof. Hopefully in the near future, we can create a geometric figure that illustrates this proof and finally uncover a geometric proof of equation (3).

I, personally, am having trouble visualizing the proof into a picture. At this time, I do not know how to make the connection. I still cannot create an figure that includes all of the angles needed in the identity represented by equation (3).

# 6 Week of April 30, 2007

## 6.1 Continuing with the Trigonometric Identity with Three Variables

During our meeting with Dr. Mohlenkamp, we examined a figure in terms of two variables. One variable represented $x+y$ and the other represented $z$. We were able to prove equation (1) again, but this did not help us with equation (3). We left with ambitions to extend from this work. When we undergraduates met on Friday, we continued to try and find a figure. I think that we lack the tools to look at this problem from another viewpoint; it appears that we are beating this horse to death. We have drawn every figure possible. We did create a new figure. I have yet to look at the application of equation (2), the Law of Sines, to this new figure. We will see if this leads to any promising proof.

# 7 Week of May 7, 2007

## 7.1 Continuing with the Trigonometric Identity with Three Variables

We three undergraduates have attempted to look at this identity from other directions. After Friday's meeting, we decided that the answer to the proof of equation (3) might include using equation (1), $\sin(A+B)$. I have attempted to look at this identity from this angle, but I am struggling to develop anything substantial. My next thought is to work backwards from the end result to see if we can discover what we need in the Algebraic steps of the proof and then translating this to a picture.

# 8 Week of May 14, 2007

## 8.1 Progress with the Trigonometric Identity with Three Variables

As before mentioned, we were able to use equation (1) and Algebra to expand $\sin(A+B+C)$ into the trigonometric identity for which we were aiming. However, the geometric picture is not clear. It is difficult to develop a picture in which the free parameters are independent and all of the angles within the identity from equation (3) are present. I also attempted to produce an identity of $\sin(A+B+C+D)$ in a similar fashion, using equation (1), Algebra and Trigonometry. The computation of this is quite messy. The other two undergraduates attempted to construct a figure with four angles — $A$, $B$, $C$, and $D$. However, I believe this seemed more difficult than with three angles.

I have personally worked on Geometer's Sketchpad in attempts to create a plausible figure for three angles. I was able to create a figure but am missing a few angles. However, the trigonometric identity did check with the figure I created. An interesting fact that I thought was important was the idea that the intersection of the lines determined by the angles $\phi$ and $\theta$ created a vertical angle with measure $\phi - \theta$.

# 9 Week of May 21, 2007

## 9.1 Trigonometric Identity with Three Variables: An Unsolved Mystery

In this last week, we were unable to make any substantial progress. We are still unable to construct a figure that encompasses all of the angles needed in the identity. We continued to explore $\sin(A+B+C+D)$ using previous identities, Algebra and Trigonometry. However, the computations become very complicated. We will concede defeat to this problem and prepare a presentation for the other group members to discuss our work over the past nine weeks.

## References

[1] Mohlenkamp, Martin J. and Lucas Monzón. "Trigonometric Identities and Sums of Separable Functions." The Mathematical Intelligencer 27.2 (2005): 65-69.

[2] Nelson, Roger B. Proof Without Words: Exercises in Visual Thinking. Washington, D.C.: The Mathematical Association of America, 1993.