1 09/10/2014: Perturbations of orthogonal projections from perturbations of subspaces

I have completed the part of my bound on perturbations of orthogonally projected solution which I had anticipated being potentially difficult. For now, I am working under the assumption that I have a real inner product. Once the proof is complete, I will attempt to generalize to the complex case.
I have nearly completed the RCR course (completion was delayed when I temporarily lost internet access).
The autobiography is written, but entirely too Spartan in its current state.

2 09/17/2014: Perturbations, continued. Initial approach failing.

This week's goal was to bound the perturbations of the denominator of the condition number. Bounds resulting from the perturbation of the summands are easier to create, but may be quite weak if the approximation is ill-conditioned. If results from numerical linear algebra can be adapted to this problem, the bound might possibly be made independent of the conditioning of the problem.
Over the past week, I have attempted to find (lower) bounds on $\mathrm{Re}(\frac{\langle x_U, x_V \rangle}{\|x_U\| \|x_V\|})$, where x_U, x_V are orthogonal projections of x onto subspaces U, V respectively, based on the angle between U and V, defined as $\cos^{-1}(\inf_{u \in U} \sup_{v \in V} \langle u, v \rangle)$. So far, I have been unsuccessful.
My current plan is to use the characterization of x_U as the vector in $U \cap S(x/2, \|x\|/2)$ such that $\|x_U\|$ is maximized.

3 09/24/2014: Generalization begins

I have been unable to complete the bound I was working on last week. Fortunately, this bound is not strictly necessary, as a different bound will suffice, though there remains a possibility that it might improve the bound on condition number, especially in the event that two vectors destructively interfere.
After some time attempting to modify the definitions I was originally using, I have decided to abandon them, if possible, as they have proven difficult to modify and prevent progress toward removing the dependency on basis.

4 10/01/2014: Generalization continues

I have abandoned my old definitions completely, instead using definitions from Musings on Multilinear Fitting. Some definitions correspond exactly to the ones I was using earlier; some do not. In particular, I need to be very careful when replacing A^*T by $\mathbf{b}$, as much of my previous work applies only pointwise.

That said, it has become apparent that working in a Hilbert space makes the techniques of linear algebra invaluable.

For explaining the change between G_1 and G_2, I believe that $\|G_1 - G_2\|$ and $\kappa(G_1) - \kappa(G_2)$ are sufficient for measuring changes when $r = 2$. For $r > 2$, other measures of the angles between separable summands should be sufficient for measuring internal differences. This has not been confirmed, however.

5 10/08/2014: Generalization almost complete

The upper bound on which I've been working has been adapted to the form required by Musings on Multilinear Fitting. I've split the proofs further, in the interest of having more generally useful results, and have removed all big-O notation. It has become clear that I should have done this earlier, as combining theorems (in an attempt to control the complexity of the bound) led me to inadvertently reinvent the wheel. On the bright side, the theorem created here does not rely on big-O notation, unlike that found in the literature.
I do not yet have an elegant means of relating a pointwise bound to a bound in the function's norm; this will prevent any truly interesting results until resolved.

6 10/15/2014: Generalized bound achieved

The pointwise bound allowed a bound using the function norm, through integration. This does restrict which norms may be used, as I had originally wished to permit any norm induced by an inner product, but the results obtained by restriction are far more useful. In the future, I must more seriously consider reducing the scope of problems so that solutions may be found.
I have achieved a bound on the numerator of the condition number, although it has an unfortunately difficult prerequisite that may prevent a useful bound.

7 10/22/2014: Bound improved, thoughts

An error has been corrected, the unfortunate prerequisite has been removed, and some investigation permitted other requirements to be weakened.

All necessary bounds have been established, but I have not yet had the opportunity to combine them. At this point, I am unsure of the degree to which I will be able to simplify the final bound.
To improve clarity, I have rearranged and better documented some theorems, altered my notation, and added a bibliography. I now employ the hyperref LaTeXpackage.

8 10/29/2014: Not much progress

The bounds have been combined into a bound on the L_2 condition number, but the bound is far from elegant. The same methods may be employed to create a bound on the L_1 condition number, though if one bound does not provide insight, the other is similarly unlikely to do so. That said, if it is complexity that obscures insight rather than a loose bound, then the L_1 bound may yet be worthwhile.

9 11/5/2014: Bound improved

I've combined some of the variables I'd been used, and I have further simplified (and improved) the bound. Initial results indicate that the common wisdom about condition number may be substantiated, unlike our previous assumptions. However, the expression is still too complicated to provide immediate insight.
At this point it is difficult to judge the utility of finding an L_1 bound; it would be simpler than the L_2 bound, but would suffer from many of the same problems, including a dependence on both the norm of the target and the norm of the summands of the approximation.

10 11/12/2014: Laplacian

The error when fitting the Laplacian target indicates multiple local minima, including the target, each having $\alpha = -\beta$. This structure appears independent of the number of dimensions.

11 11/19/2014: Laplacian (continued)

I've identified exact locations of all local minima of the error function for the symmetric rank-2 case (after removing discontinuities). Some regions approaching the minima occurring at $\alpha = \beta = \pm 2 \tan^{-1}\left(\sqrt{d - 1 - \sqrt{d(d-2)}}\right)$ demonstrate constructive interference (L_2 condition number < 1). Should we desire, we will be able to construct sequences approaching these local minima, having either orthogonality or L_2 condition number < 1.

I've examined the error landscape of rank-1 approximations of the Laplacian target. Due to a false start, examination of the ALS update function for this target has been delayed.