score	possible	page
	25	1
	30	2
	20	3
	25	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/15 1. Find the equation in general form for the plane that passes through the points $P = (1, -1, 1)$, $Q = (2, 2, 1)$, and $R = (2, 1, 2)$. [similar to 11.6 # 9] Select two vectors in the plane, such as

$$\vec{u} = \overrightarrow{PQ} = \langle 2, 2, 1\rangle - \langle 1, -1, 1\rangle = \langle 1, 3, 0\rangle$$
$$\vec{v} = \overrightarrow{RQ} = \langle 2, 2, 1\rangle - \langle 2, 1, 2\rangle = \langle 0, 1, -1\rangle.$$

Compute a normal vector

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 0 \\ 0 & 1 & -1 \end{vmatrix} = (-3\vec{i} + 0\vec{j} + 1\vec{k}) - (0\vec{k} - 1\vec{j} + 0\vec{i}) = -3\vec{i} + 1\vec{j} + 1\vec{k} = \langle -3, 1, 1\rangle.$$

Using P as the base point, an equation in standard form is

$$0 = \vec{n} \cdot (\vec{x} - \overrightarrow{0P}) = -3(x - 1) + 1(y + 1) + 1(z - 1)$$

so the equation in general form is

$$-3x + y + z = -3.$$

/10 2. Calculate $\int \langle t\sin(t^2), te^t\rangle\, dt$. [similar to 12.2 #33] For the first integral, use substitution with $u = t^2 \Rightarrow du = 2tdt$ to get

$$\int t\sin(t^2)dt = \int \frac{1}{2}\sin(u)du = -\frac{1}{2}\cos(u) + C = -\frac{1}{2}\cos(t^2) + C.$$

For the second integral, use integration by parts with $u = t$ and $dv = e^t dt$ to get

$$\int te^t dt = te^t - \int e^t dt = te^t - e^t + C.$$

Combined, we have $\langle -\frac{1}{2}\cos(t^2), te^t - e^t\rangle + \vec{C}$.

3. Let $\vec{r}(t) = \langle \sin(t), 3\cos(2t) \rangle$.

/5 (a) Compute its derivative $\vec{r}'(t)$. [similar to 12.2 #12]

$$\vec{r}'(t) = \langle \cos(t), -6\sin(2t) \rangle$$

/5 (b) Compute its unit tangent vector $\vec{T}(t)$. [similar to 12.4 #5]

$$\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{\langle \cos(t), -6\sin(2t) \rangle}{\sqrt{\cos^2(t) + 36\sin^2(2t)}}$$

/5 (c) Find an equation for the tangent line to the graph of $\vec{r}(t)$ at $t = \pi/4$. [similar to 12.2 #24]

$\ell(t) = \vec{r}(\pi/4) + t\vec{r}'(\pi/4) = \langle \sin(\pi/4), 3\cos(\pi/2) \rangle + t\langle \cos(\pi/4), -6\sin(\pi/2) \rangle = \langle 1/\sqrt{2}, 0 \rangle + t\langle 1/\sqrt{2}, -6 \rangle$

One could use $\vec{T}(\pi/4)$ instead of $\vec{r}'(\pi/4)$ and get

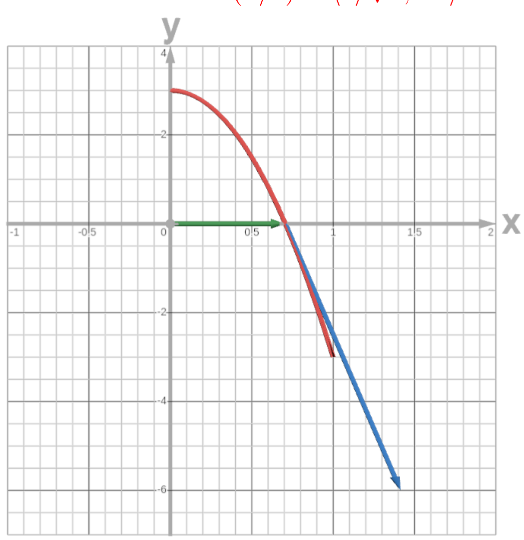
$$\ell(t) = \vec{r}(\pi/4) + tT(\pi/4) = \langle \sin(\pi/4), 3\cos(\pi/2) \rangle + t\frac{\langle \cos(\pi/4), -6\sin(\pi/2) \rangle}{\sqrt{\cos^2(\pi/4) + 36\sin^2(\pi/2)}}$$

$$= \langle 1/\sqrt{2}, 0 \rangle + t\frac{\langle 1/\sqrt{2}, -6 \rangle}{\sqrt{(1/2) + 36}} = \langle 1/\sqrt{2}, 0 \rangle + t\left\langle 1/\sqrt{73}, -6\sqrt{2/73} \right\rangle.$$

/5 (d) Set up, but do not evaluate, the integral that would give the arclength of $\vec{r}(t)$ on the interval $[0, \pi]$. [similar to 12.2 #41]

$$\int_0^{\pi} \|\vec{r}'(t)\| dt = \int_0^{\pi} \sqrt{\cos^2(t) + 36\sin^2(2t)} dt.$$

/10 (e) Sketch $\vec{r}(t)$ on the interval $[0, \pi]$. Add and label the vector $\vec{r}(\pi/4)$. Add and label the vector $\vec{r}'(\pi/4)$ starting at $\vec{r}(\pi/4)$.
[similar to 12.2 #17] On $[0, \pi]$, the x-coordinate $\sin(t)$ goes from 0 to 1 and back to 0, while the y-coordinate $3\cos(2t)$ goes from 3 to -3 and back to 3. The green vector is $\vec{r}(\pi/4) = \langle 1/\sqrt{2}, 0 \rangle$ and the blue vector is $\vec{r}'(\pi/4) = \langle 1/\sqrt{2}, -6 \rangle$.



/6 4. (a) Calculate $\lim_{t\to 0}\left\langle \frac{t}{\sin(t)}, (1+t)^{1/t}\right\rangle$. [12.2 #7] Both of these are special limits,

$$\lim_{t\to 0}\left\langle \frac{t}{\sin(t)}, (1+t)^{1/t}\right\rangle = \langle 1, e\rangle$$

(b) Consider the limit $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2)}{y}$. [13.2 #18]

/5 i. Evaluate the limit along the path $y = mx$. Using L'Hôpital's rule,

$$\lim_{x\to 0}\frac{\sin(x^2)}{mx} = \lim_{x\to 0}\frac{\cos(x^2)2x}{m} = \frac{\cos(0)0}{m} = 0.$$

/5 ii. Evaluate the limit along the path $y = x^2$.

$$\lim_{x\to 0}\frac{\sin(x^2)}{x^2} = \lim_{t\to 0^+}\frac{\sin(t)}{t} = 1.$$

/4 iii. From your results above, can we conclude that the limit exits or that it does not exit? Explain you reasoning. Since we found that the limit is different along different paths, the 2-dimensional limit does not exist. (If the limits along different paths had agreed, that would not be enough to conclude the limit existed.)

5. Let $f(x,y) = \dfrac{1}{\sqrt{y^2 - x^2}}$.

/6 (a) What is the domain of f? Is it open, closed, or neither? [similar to 13.2 #13] To avoid taking the root of a negative number or dividing by 0, we need $y^2 - x^2 > 0$, which is equivalent to $\{(x,y) \ : \ |x| < |y|\}$. Since we have strict inequality, the set is open.

/4 (b) What is the range of f? Explain your reasoning. The denominator is always real and positive, and can be made arbitrarily large or small by setting $x = 0$ and varying y. Thus the range is $(0, \infty)$.

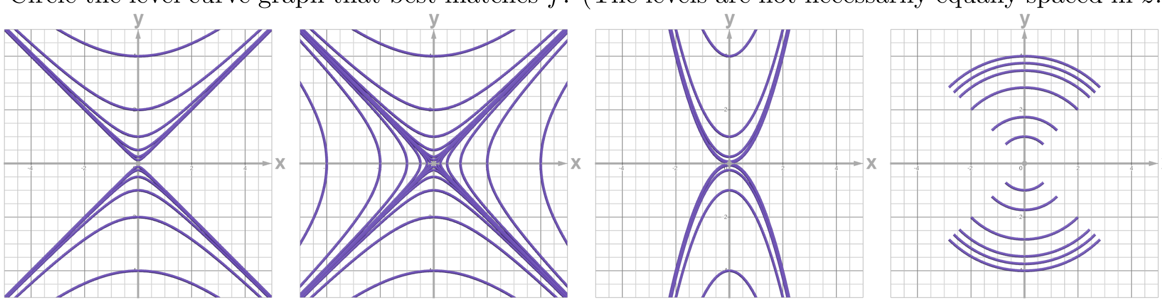
/5 (c) Calculate f_x. [similar to 13.3 #26]

$$f_x(x,y) = \frac{-1}{2}(y^2 - x^2)^{-3/2}(-2x) = \frac{x}{(y^2 - x^2)^{3/2}}$$

/5 (d) Calculate f_{xy}. [similar to 13.3 #26]

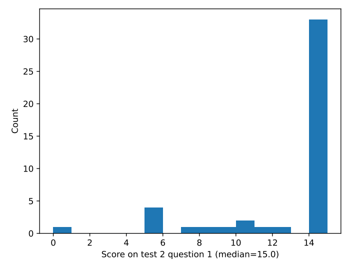
$$f_{xy}(x,y) = x\frac{-3}{2}(y^2 - x^2)^{-5/2}(2y) = \frac{-3xy}{(y^2 - x^2)^{5/2}}$$

/5 (e) Circle the level curve graph that best matches f. (The levels are not necessarily equally spaced in z.)



[similar to 13.1 #19] The first and third graphs have the correct domain. As we approach the lines $|x| = |y|$, the function goes to infinity, so there should be level curves following these lines, which the first and second graphs have. Thus it can only be the first graph.

Scores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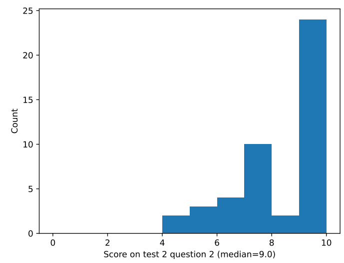
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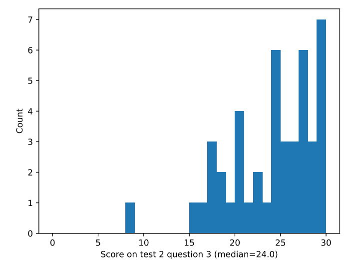
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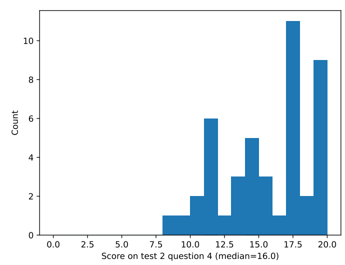
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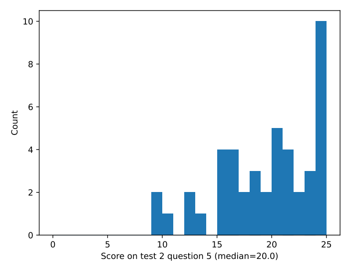
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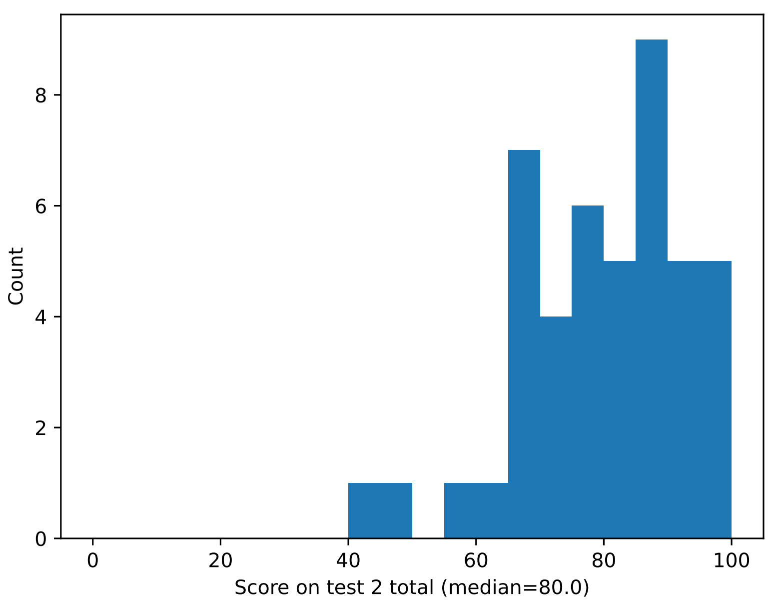
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Score on test 2 total (median=80.0)