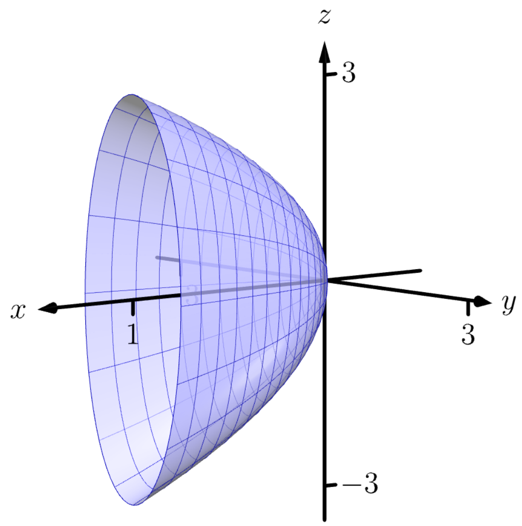
score	possible	page
	16	1
	29	2
	25	3
	30	4
	100	

Name: ______________________________

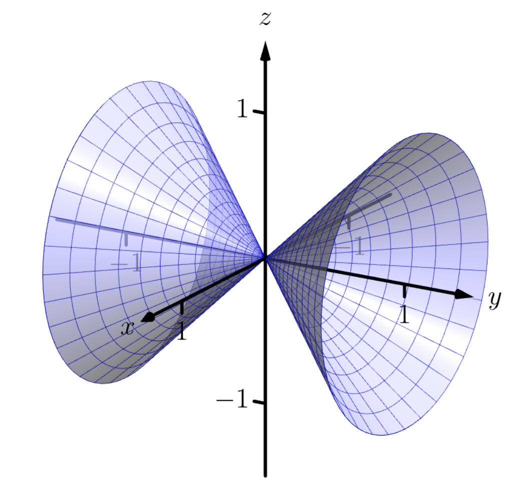
Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/16 1. For each graph, circle the equation that best fits it.



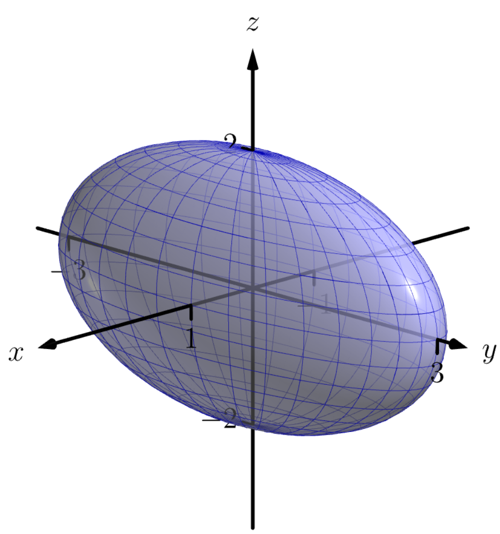
$\boxed{x = y^2 + \frac{z^2}{9}}$ or $x = y^2 + \frac{z^2}{3}$

[11.1 # 23] since it includes $(1, 0, 3)$



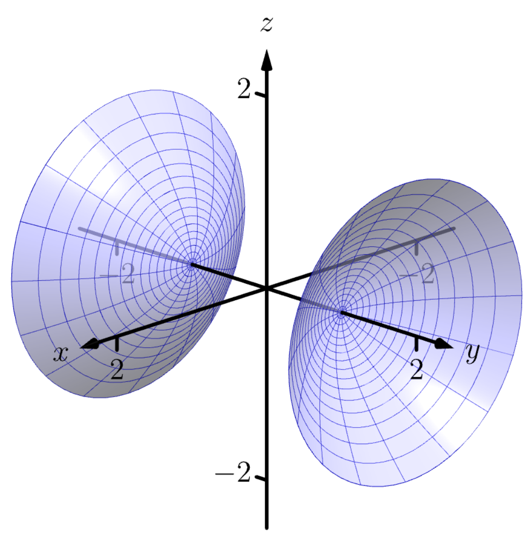
$x^2 - y^2 - z^2 = 0$ or $\boxed{x^2 - y^2 + z^2 = 0}$

[11.1 # 24] since $y = 1$ gives a circle



$x^2 + \frac{y^2}{3} + \frac{z^2}{2} = 1$ or $\boxed{x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1}$

[11.1 # 25] since includes $(1, 0, 0)$, $(0, 3, 0)$, $(0, 0, 2)$

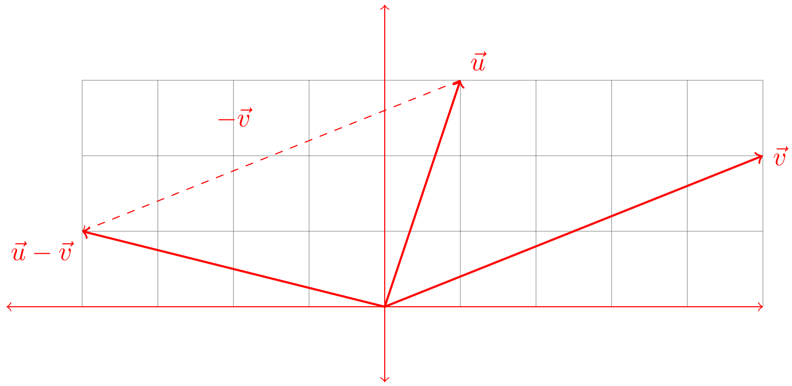


$\boxed{y^2 - x^2 - z^2 = 1}$ or $y^2 + x^2 - z^2 = 1$

[11.1 # 26] since does not include $(0, 1, 0)$

2. Let $\vec{u} = \langle 1, 3\rangle$ and $\vec{v} = \langle 5, 2\rangle$.

/8 (a) Graph and label $\vec{u}$, $\vec{v}$, and $\vec{u} - \vec{v}$. [similar to 11.2 #11]



/7 (b) Construct the unit vector in the direction of $\vec{u}$. [similar to 11.2 # 22]

$$\|\vec{u}\| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

so the unit vector is

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{\langle 1, 3\rangle}{\sqrt{10}} = \langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\rangle.$$

/8 (c) Compute the angle between $\vec{u}$ and $\vec{v}$ in radians. Since you do not have a calculator, leave you answer as an ugly expression $\theta = \ldots$. [similar to 11.3 #13] Computing

$$\|\vec{v}\| = \sqrt{5^2 + 2^2} = \sqrt{29} \text{ and}$$
$$\vec{u} \cdot \vec{v} = 5 + 6 = 11,$$

the angle is

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}\right) = \cos^{-1}\left(\frac{11}{\sqrt{10}\sqrt{29}}\right).$$

/6 (d) Find the orthogonal projection of $\vec{u}$ onto $\vec{v}$ (which is denoted $\text{proj}_{\vec{v}}\vec{u}$). [similar to 11.3 #24]

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}\vec{v} = \frac{11}{29}\langle 5, 2\rangle = \langle \frac{55}{29}, \frac{22}{29}\rangle.$$

3. Let $\vec{u} = \langle 0, 1, 2\rangle$, $\vec{v} = \langle 3, 5, 7\rangle$, and $\vec{w} = \langle 1, 2, -2\rangle$.

/7 (a) Compute $\|\vec{u} - \vec{v}\|$. [similar to 11.2 # 18]

$$\vec{u} - \vec{v} = \langle 0, 1, 2\rangle - \langle 3, 5, 7\rangle = \langle -3, -4, -5\rangle,$$

so

$$\|\vec{u} - \vec{v}\| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2} = \sqrt{9 + 16 + 25} = \sqrt{50}.$$

/5 (b) Compute $\vec{u} \cdot \vec{v}$. [similar to 11.3 # 7]

$$\vec{u} \cdot \vec{v} = \langle 0, 1, 2\rangle \cdot \langle 3, 5, 7\rangle = 0 + 5 + 14 = 19.$$

/8 (c) Compute $\vec{u} \times \vec{v}$. [similar to 11.4 # 7]

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 2 \\ 3 & 5 & 7 \end{vmatrix} = (7\vec{i} + 6\vec{j} + 0\vec{k}) - (3\vec{k} + 0\vec{j} + 10\vec{i}) = -3\vec{i} + 6\vec{j} - 3\vec{k} = \langle -3, 6, -3\rangle.$$

/5 (d) Find the volume of the parallelepiped defined by $\vec{u}$, $\vec{v}$, and $\vec{w}$. [similar to 11.4 # 33] The volume is computed by the absolute value of the triple scalar product

$$|(\vec{u} \times \vec{v}) \cdot \vec{w}| = |\langle -3, 6, -3\rangle \cdot \langle 1, 2, -2\rangle| = |-3 + 12 + 6| = 15.$$

/10 4. (a) Write the vector and parametric equations for the line that passes through the point $P = (3, -5, 5)$ and is parallel to $\vec{d} = \langle 1, 2, 1\rangle$. [similar to 11.5 # 5] The vector equation is

$$\vec{\ell}(t) = \overrightarrow{0P} + t\vec{d} = \langle 3, -5, 5\rangle + t\langle 1, 2, 1\rangle,$$

so the parametric equations are

$$\begin{aligned} x &= 3 + t \\ y &= -5 + 2t \text{ and} \\ z &= 5 + t\,. \end{aligned}$$

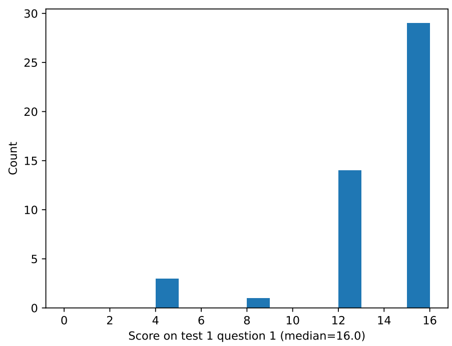
/20 (b) Find the distance from the point $Q = (3, -3, 4)$ to this line. [similar to 11.5 # 23] We compute

$$\begin{aligned} \overrightarrow{PQ} &= \langle 3, -3, 4\rangle - \langle 3, -5, 5\rangle = \langle 0, 2, -1\rangle \\ \overrightarrow{PQ} \times \vec{d} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (2\vec{i} - 1\vec{j} + 0\vec{k}) - (2\vec{k} + 0\vec{j} - 2\vec{i}) = 4\vec{i} - 1\vec{j} - 2\vec{k} = \langle 4, -1, -2\rangle. \\ \|\overrightarrow{PQ} \times \vec{d}\| &= \sqrt{4^2 + (-1)^2 + (-2)^2} = \sqrt{21} \\ \|\vec{d}\| &= \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6} \end{aligned}$$

and then use the distance formula

$$\frac{\|\overrightarrow{PQ} \times \vec{d}\|}{\|\vec{d}\|} = \frac{\sqrt{21}}{\sqrt{6}}.$$

Scores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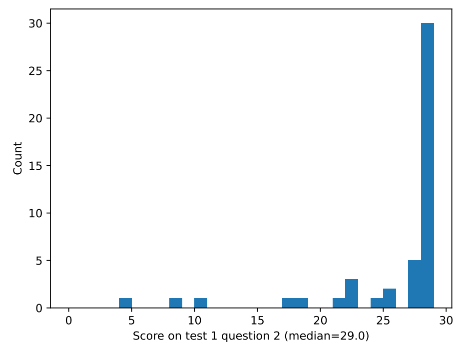
Score on test 1 question 1 (median=16.0)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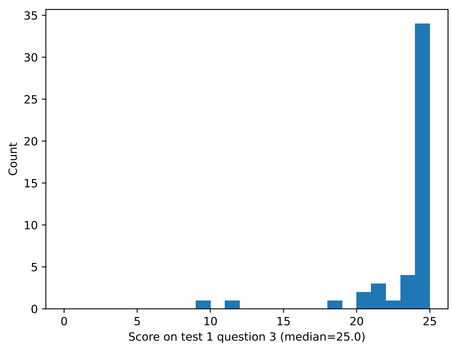
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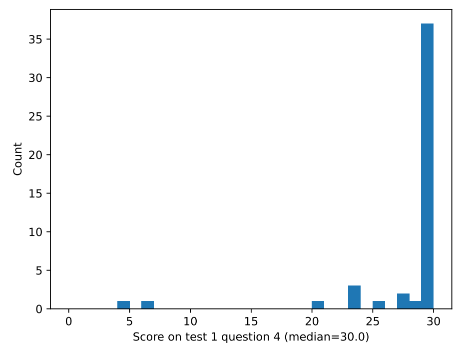
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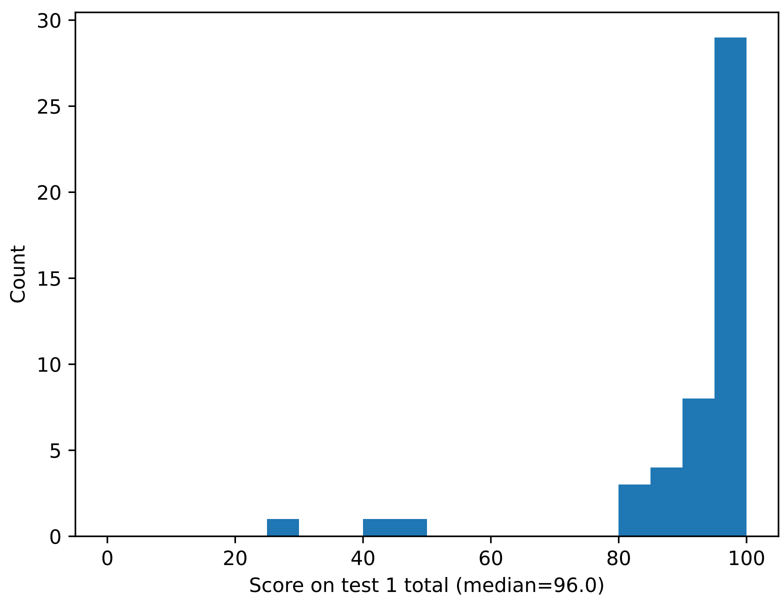
Score on test 1 question 4 (median=30.0)


Count
Score on test 1 total (median=96.0)