score	possible	page
	24	1
	20	2
	30	3
	26	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Sketch the graph of a single function f that satisfies the following conditions.

/2 (a) If $0 < x < 2$, then $f'(x) = 0$.

/2 (b) $f(1) = 2$

/2 (c) $\lim_{x \to 2^+} f(x) = 3$

/2 (d) If $2 < x < 4$, then $f'(x) = -1$.

/2 (e) f is not continuous at $x = 4$.

/2 (f) If $4 < x < 6$, then $f'(x) > 0$.

/2 (g) $f'(6) = 0$.

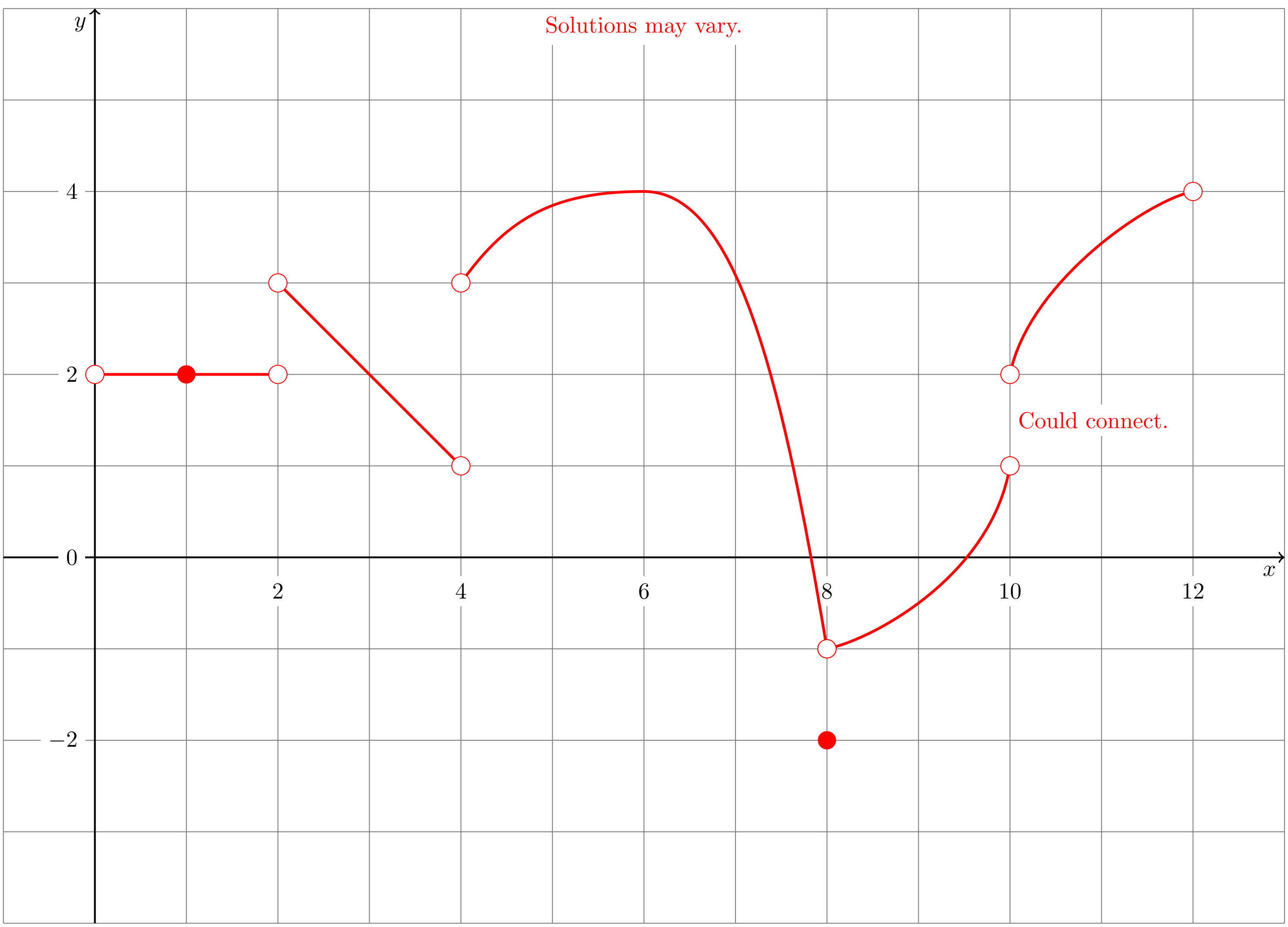
/2 (h) If $6 < x < 8$, then $f'(x) < 0$.

/2 (i) $\lim_{x \to 8} f(x) = -1$.

/2 (j) $f(8) = -2$

/2 (k) If $8 < x < 10$, then $f'(x) > 0$ and $f''(x) > 0$.

/2 (l) If $10 < x < 12$, then $f'(x) > 0$ and $f''(x) < 0$.



/10 2. [2.6 #21] Find $\frac{dy}{dx}$ using implicit differentiation for

$$x^2 \tan(y) = 200\,.$$

Differentiating both sides with respect to x yields

$$2x\tan(y) + x^2\sec^2(y)\frac{dy}{dx} = 0\,.$$

Gathering terms with $\frac{dy}{dx}$ to one side yields

$$x^2\sec^2(y)\frac{dy}{dx} = -2x\tan(y)\,.$$

Solving for $\frac{dy}{dx}$ then yields

$$\frac{dy}{dx} = \frac{-2x\tan(y)}{x^2\sec^2(y)} = \frac{-2\sin(y)\cos(y)}{x}\,.$$

/10 3. [2.6 #37, without tangent line] Find $\frac{dy}{dx}$ using logarithmic differentiation for

$$y = (1+x)^{1/x}\,.$$

Applying ln to both sides and using properties of logarithms yields

$$\ln(y) = \frac{1}{x}\ln(1+x)\,.$$

Differentiating both sides with respect to x yields

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}\ln(1+x) + \frac{1}{x}\frac{1}{1+x}\,.$$

Solving for $\frac{dy}{dx}$ then yields

$$\frac{dy}{dx} = (1+x)^{1/x}\left(-\frac{1}{x^2}\ln(1+x) + \frac{1}{x}\frac{1}{1+x}\right)\,.$$

/10 4. [3.1 #17] Find the extreme values of the function $f(x) = x^2 + 2x - 8$ on the interval $[-2, 1]$.

Differentiating yields $f'(x) = 2x + 2$. Setting $2x + 2 = 0$ and solving yields the critical number $x = -1$, which is in the interval $[-2, 1]$. There are no places where $f'(x)$ does not exist. Plugging in our the critical number and endpoints gives

$$f(-2) = (-2)^2 + 2(-2) - 8 = -8\,, \quad f(-1) = (-1)^2 + 2(-1) - 8 = -9\,, \quad \text{and} \quad f(1) = 1^2 + 2(1) - 8 = -5\,.$$

Thus the maximum value is -5 and the minimum value is -9.

/10 5. [3.4 #17] For the function $f(x) = x^3 - 7x + 9$, find the intervals where it is concave up, the intervals where it is concave down, and any inflection points.

Differentiating yields $f'(x) = 3x^2 - 7$. Differentiating a second time gives $f''(x) = 6x$. Setting $f''(x) = 0$ we find $x = 0$ is the only possible location of an inflection point.

For the interval $(-\infty, 0)$ we can check that $f''(x) < 0$, so f is concave down. For the interval $(0, \infty)$ we can check that $f''(x) > 0$, so f is concave up.

Thus there is an inflection point at $(0, f(0)) = (0, 9)$.

6. Compute the following derivatives:

/5 (a) $\dfrac{d}{dx} \tan^{-1}(\log_2(x)) =$

$$\frac{1}{1 + (\log_2(x))^2} \frac{1}{x \ln(2)}$$

/5 (b) $\dfrac{d}{dx} \csc^{-1}(x) \sec(x) =$

$$-\frac{1}{|x|\sqrt{x^2 - 1}} \sec(x) + \csc^{-1}(x) \sec(x) \tan(x)$$

7. [3.5 #17] For the function $f(x) = (x-2)^2 \ln(x-2)$:

   - Assume that f has no vertical asymptotes and that $\lim_{x \to 2^+} f(x) = 0$.

/2 (a) Find the domain.

/2 (b) Find any horizontal asymptotes (or say there are none).

/4 (c) Find the critical values.

/4 (d) Find the intervals on which f is increasing or decreasing.

/3 (e) Find the local maximum and minimum values of f.

/3 (f) Find the intervals on which f is concave up or concave down.

/3 (g) Find the inflection points.

/5 (h) Use the information above to sketch the graph.

Since the domain of $\ln(x)$ is $x > 0$, the domain of f is $x > 2$.

We cannot take the limit $x \to -\infty$. Since both x^2 and $\ln(x)$ increase as x increases, $\lim_{x\to\infty} f(x) = \infty$ so there are no horizontal asymptotes.

We can compute

$$f'(x) = 2(x-2)\ln(x-2) + (x-2)^2 \frac{1}{x-2} = (x-2)(2\ln(x-2)+1)\,.$$

Setting $f'(x) = 0$ then gives $x - 2 = 0$ or $2\ln(x-2) + 1 = 0$. The first would give $x = 2$, but that is outside the domain. Solving the second gives $x = 2 + e^{-1/2}$, which is our critical value.

We can compute

$$f''(x) = (2\ln(x-2)+1) + (x-2)\frac{2}{x-2} = 2\ln(x-2) + 3\,.$$

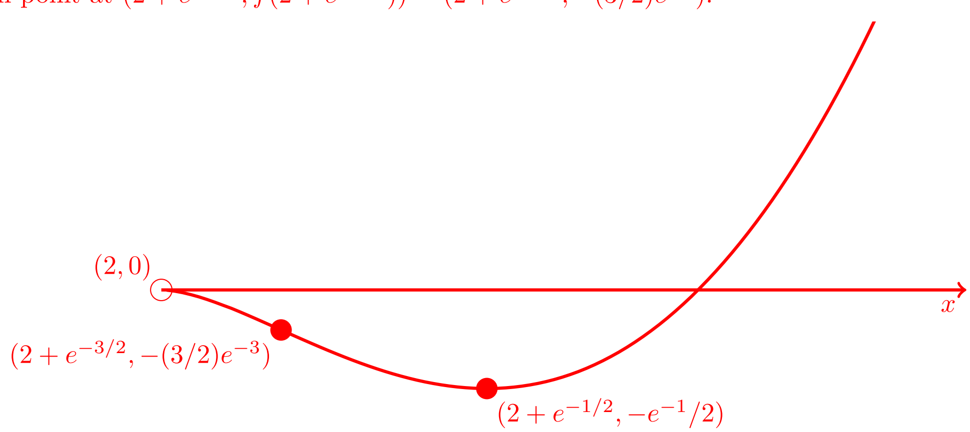
Setting $f''(x) = 0$ then gives the only possible location of a inflection point at $x = 2 + e^{-3/2}$.

Assembling into a chart and checking signs, we have

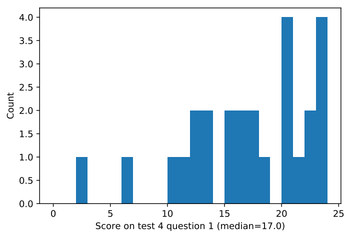
f		I.P.		min	
f''	$-$	0	$+$	$+$	$+$
f'	$-$	$-$	$-$	0	$+$
	$(2, 2+e^{-3/2})$	$2+e^{-3/2}$	$(2+e^{-3/2}, 2+e^{-1/2})$	$2+e^{-1/2}$	$(2+e^{-1/2}, \infty)$

There is a local minimum at $x = 2 + e^{-1/2}$ with value $f(2+e^{-1/2}) = e^{-1}\ln(e^{-1/2}) = -e^{-1}/2$ and no local maxima.

There is an inflection point at $(2+e^{-3/2}, f(2+e^{-3/2})) = (2+e^{-3/2}, -(3/2)e^{-3})$.



Scores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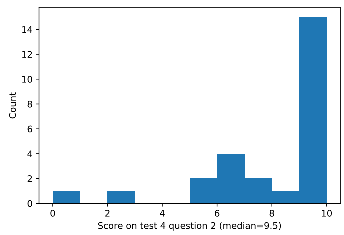
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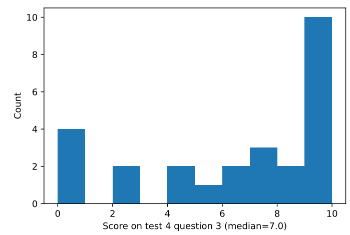
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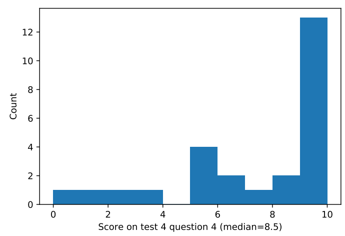
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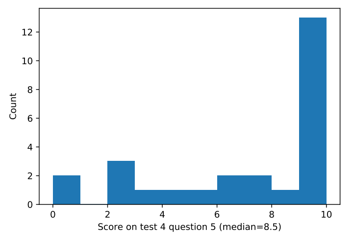
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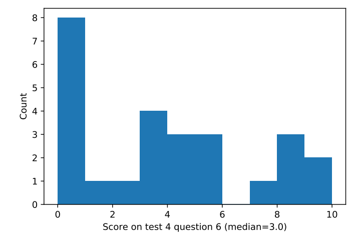
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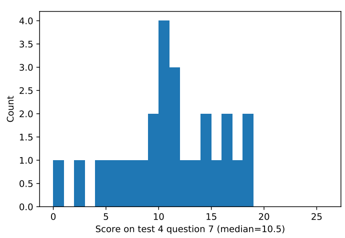
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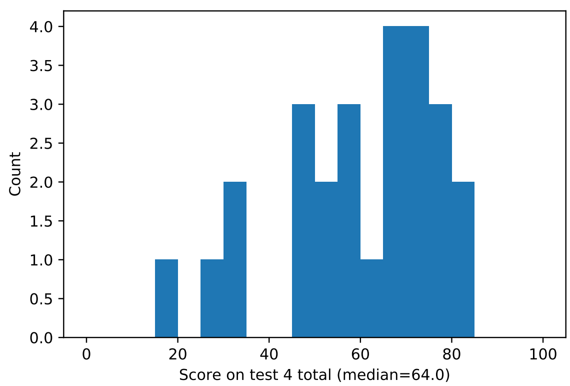
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Score on test 4 total (median=64.0)