score	possible	page
	25	1
	25	2
	26	3
	24	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Compute the following limits. Do **not** use L'Hôpital's rule.

/5 (a) $\lim_{x\to 4^+} \frac{x+3}{x-4} =$

Since $x \to 4^+$, we know $x - 4 > 0$, so we have

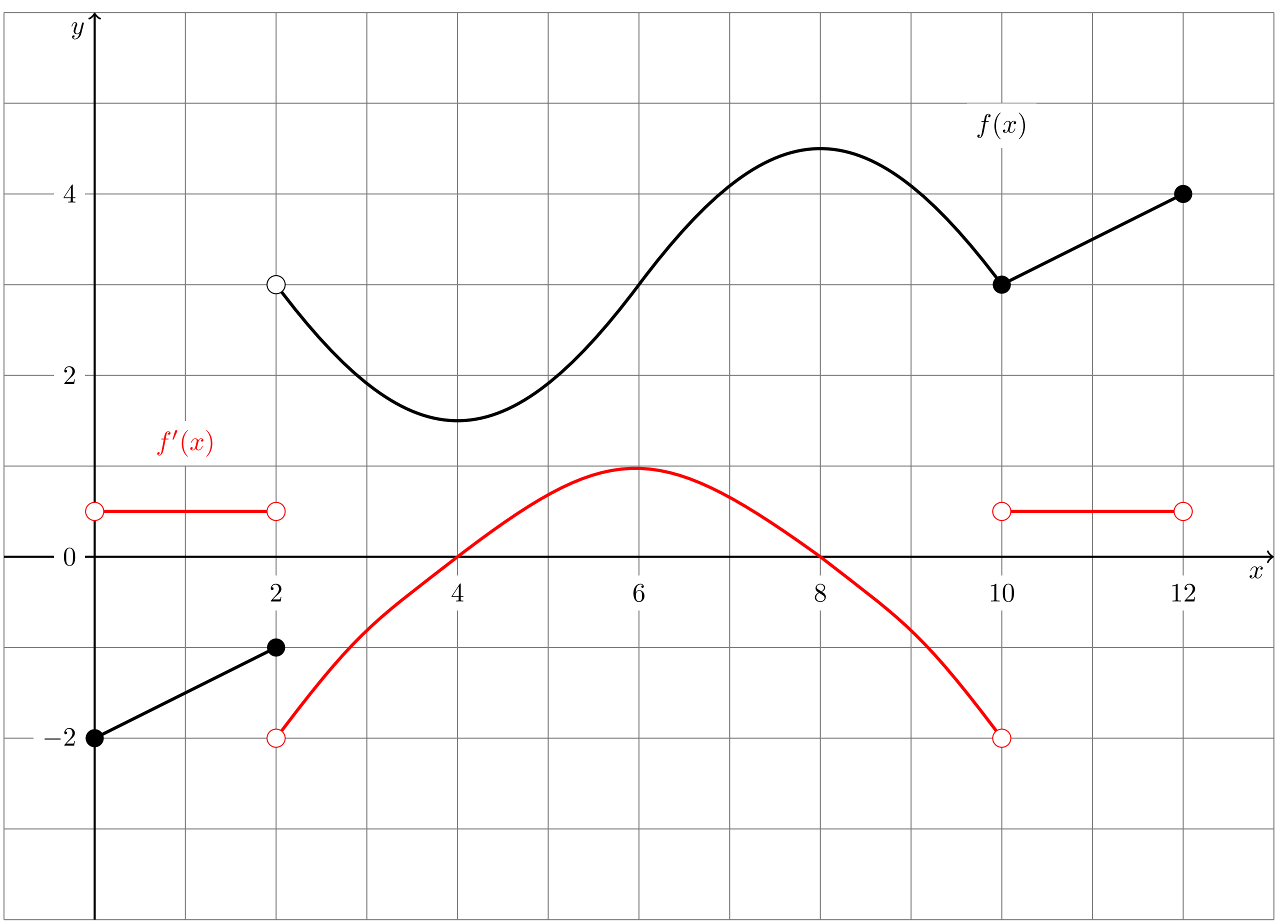
$$\lim_{x\to 4^+} \frac{7}{x-4} = \lim_{t\to 0^+} \frac{7}{t} = \infty\,.$$

/5 (b) $\lim_{x\to -\infty} \frac{5x^3 + 9x^5 + 2}{11x^5 - 13} =$

Multiplying the numerator and denominator by x^{-5} yields

$$\lim_{x\to -\infty} \frac{5x^{-2} + 9 + 2x^{-5}}{11 - 13x^{-5}} = \frac{0+9+0}{11-0} = \frac{9}{11}\,.$$

/15 2. The graph of a function f is given below. On the same axes, sketch the graph of f'.



/4 3. (a) Complete the definition of the *Derivative Function*:

Let f be a differentiable function on an open interval I. The function

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

is the derivative of f.

/6 (b) Using this definition, compute the derivative of $f(x) = 3x^2 + 7x$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(3(x^2+2xh+h^2)+7x+7h)-(3x^2+7x)}{h} \\ &= \lim_{h\to 0} \frac{6xh+3h^2+7h}{h} \\ &= \lim_{h\to 0}(6x+3h+7) \end{aligned}$$

$$\begin{aligned} &= \lim_{h\to 0} \frac{(3(x+h)^2+7(x+h))-(3x^2+7x)}{h} \\ &= \lim_{h\to 0} \frac{3x^2+6xh+3h^2+7x+7h-3x^2-7x}{h} \\ &= \lim_{h\to 0} \frac{h(6x+3h+7)}{h} \\ &= 6x+7\,. \end{aligned}$$

/4 4. (a) Complete the statement of the *Intermediate Value Theorem*:

Let f be a continuous function on $[a,b]$ and, without loss of generality, let $f(a) < f(b)$.
Then for every value y, where $f(a) < y < f(b)$,
there is at least one value c in (a,b) such that $f(c) = y$

/6 (b) Use the Intermediate Value Theorem to show that the equation $x^7 + x^2 = 4$ has a solution.

Let $f(x) = x^7 + x^2$, so we want to show a solution to $f(x) = 4$ exists. Since x^7 and x^2 are both continuous, so is $f(x)$. Plugging in, we find

$$f(0) = 0 + 0 = 0 < 4 \quad \text{and}$$
$$f(2) = 2^7 + 2^2 = 2^7 + 4 > 4\,.$$

So, by the Intermediate Value Theorem with $a = 0$, $b = 2$, and $y = 4$, there must exist $0 < c < 2$ such that $f(c) = 4$.

/5 5. Given that $f(3) = 5$ and $f'(3) = 7$, write an equation for the tangent line to the graph $y = f(x)$ at $x = 3$.

Using point-slope form, the tangent line is $y - f(a) = f'(a)(x-a)$. Plugging in the given values yields $y - 5 = 7(x-3)$.

6. Compute the following derivatives. (Use derivative rules, rather than computing the limits.)

/2 (a) $f(x) = 5 \Rightarrow f'(x) = 0$

/2 (b) $f(x) = x^9 \Rightarrow f'(x) = 9x^8$

/2 (c) $f(x) = \sin(x) \Rightarrow f'(x) = \cos(x)$

/2 (d) $f(x) = \cos(x) \Rightarrow f'(x) = -\sin(x)$

/2 (e) $f(x) = e^x \Rightarrow f'(x) = e^x$

/2 (f) $f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$

/2 (g) $f(x) = \tan(x) \Rightarrow f'(x) = \sec^2(x)$

/2 (h) $f(x) = \cot(x) \Rightarrow f'(x) = -\csc^2(x)$

/2 (i) $f(x) = \sec(x) \Rightarrow f'(x) = \sec(x)\tan(x)$

/2 (j) $f(x) = \csc(x) \Rightarrow f'(x) = -\csc(x)\cot(x)$

/2 (k) $f(x) = \frac{1}{x^2} \Rightarrow f'(x) = -2x^{-3}$

/2 (l) $f(x) = 9^x \Rightarrow f'(x) = 9^x \ln(9)$

/2 (m) $f(x) = \log_2(x) \Rightarrow f'(x) = \frac{1}{x\ln(2)}$

7. Compute the following derivatives. (Use derivative rules, rather than computing the limits.)

/5 (a) $\dfrac{d}{dx}x^3\sin(x) =$

$$\left(\frac{d}{dx}x^3\right)\sin(x)+x^3\left(\frac{d}{dx}\sin(x)\right)=3x^2\sin(x)+x^3\cos(x)$$

/5 (b) $\dfrac{d}{dt}\dfrac{3t}{2+7t^3} =$

$$\frac{\left(\frac{d}{dt}3t\right)(2+7t^3)-3t\left(\frac{d}{dt}(2+7t^3)\right)}{(2+7t^3)^2}=\frac{3(2+7t^3)-3t(7(3t^2))}{(2+7t^3)^2}$$

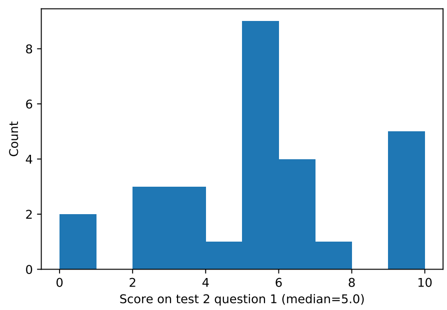
/5 (c) $\dfrac{d}{dx}\left(3x^4+\cos(x)\right)^9 =$

$$9\left(3x^4+\cos(x)\right)^8\left(\frac{d}{dx}\left(3x^4+\cos(x)\right)\right)=9\left(3x^4+\cos(x)\right)^8\left(3(4x^3)-\sin(x)\right)$$

/9 (d) $\dfrac{d}{dt}\left(7\sin\left(\dfrac{\cos(t)}{1+4^t}\right)(5t^4+3t)\right) =$

$$7\left(\left(\frac{d}{dt}\sin\left(\frac{\cos(t)}{1+4^t}\right)\right)(5t^4+3t)+\sin\left(\frac{\cos(t)}{1+4^t}\right)\left(\frac{d}{dt}(5t^4+3t)\right)\right)$$
$$=7\left(\cos\left(\frac{\cos(t)}{1+4^t}\right)\left(\frac{d}{dt}\left(\frac{\cos(t)}{1+4^t}\right)\right)(5t^4+3t)+\sin\left(\frac{\cos(t)}{1+4^t}\right)(5(4t^3)+3)\right)$$
$$=7\left(\cos\left(\frac{\cos(t)}{1+4^t}\right)\frac{(-\sin(t))(1+4^t)-\cos(t)(4^t\ln(4))}{(1+4^t)^2}(5t^4+3t)+\sin\left(\frac{\cos(t)}{1+4^t}\right)(5(4t^3)+3)\right)$$

Scores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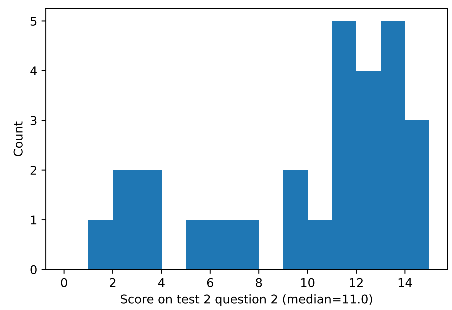
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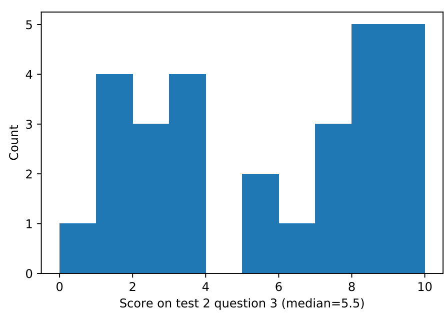
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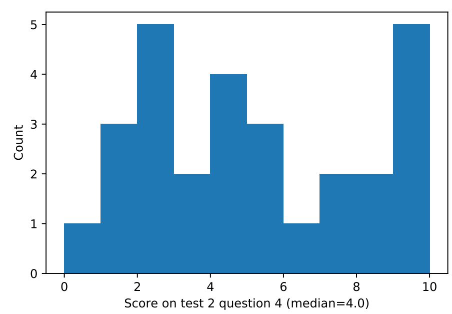
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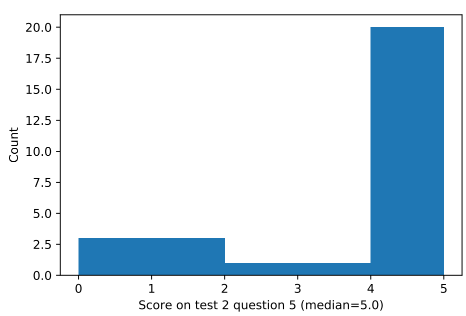
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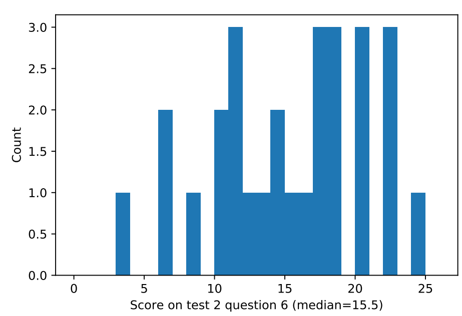
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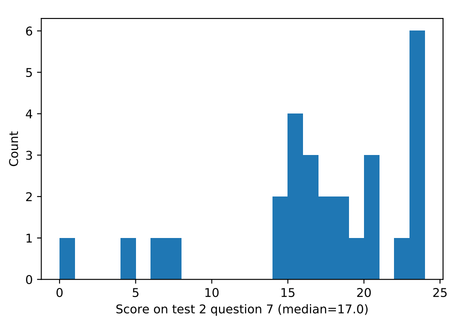
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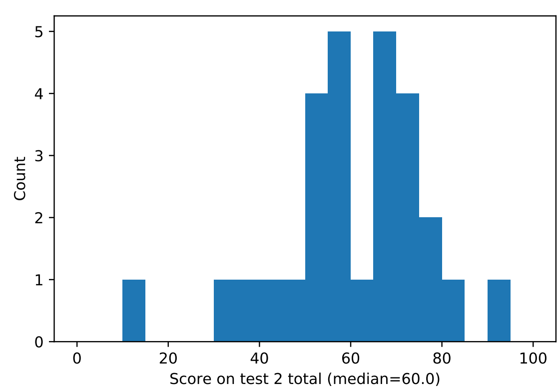
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Score on test 2 total (median=60.0)