score	possible	page
	24	1
	30	2
	25	3
	21	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Sketch the graph of a single function f that satisfies the following conditions.

/2 (a) If $0 < x < 2$, then $f'(x) = 0$.

/2 (b) $f(1) = 2$

/2 (c) $\lim_{x \to 2^+} f(x) = 3$

/2 (d) If $2 < x < 4$, then $f'(x) = -1$.

/2 (e) f is not continuous at $x = 4$.

/2 (f) If $4 < x < 6$, then $f'(x) > 0$.

/2 (g) $f'(6) = 0$.

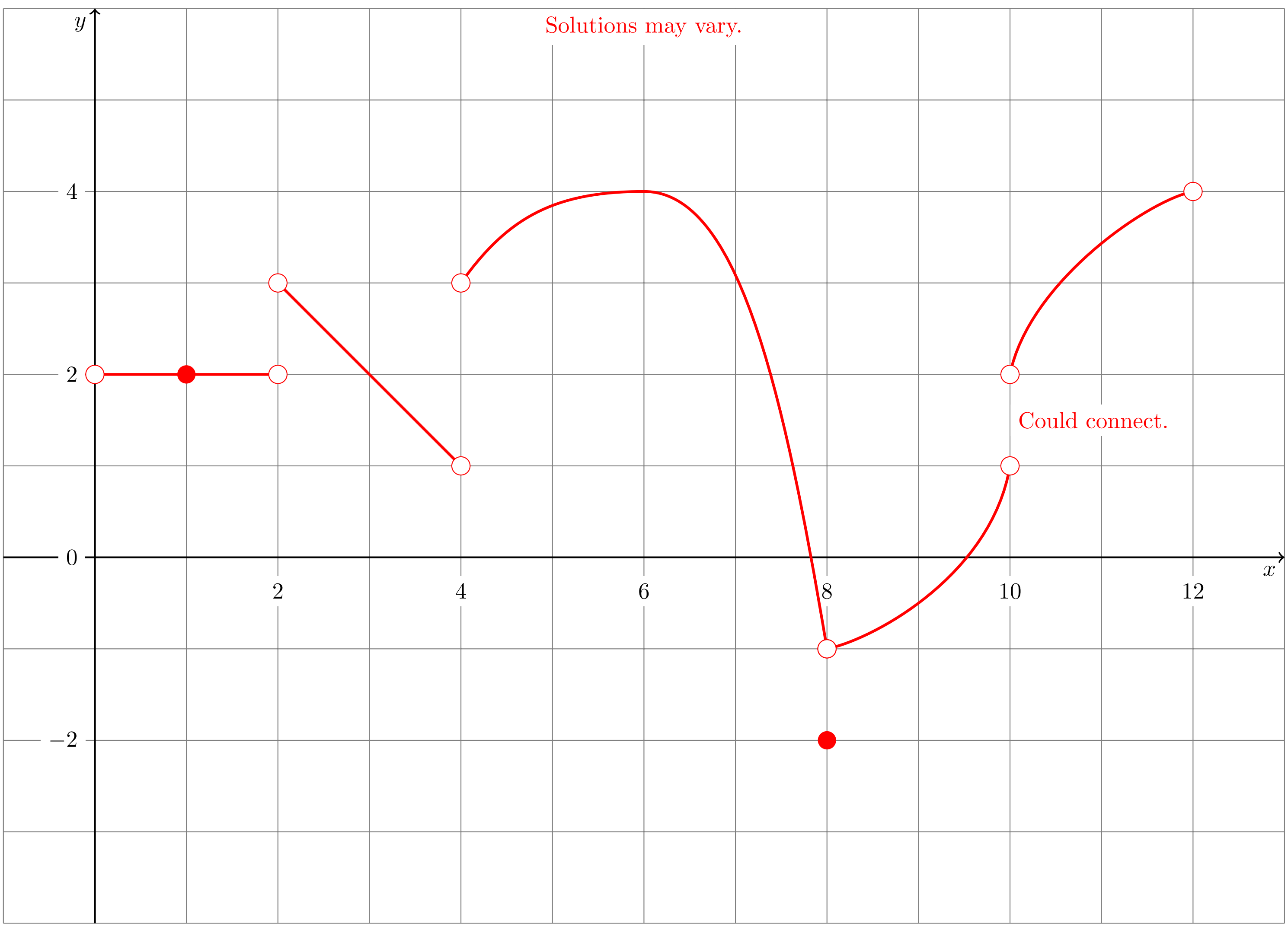
/2 (h) If $6 < x < 8$, then $f'(x) < 0$.

/2 (i) $\lim_{x \to 8} f(x) = -1$.

/2 (j) $f(8) = -2$

/2 (k) If $8 < x < 10$, then $f'(x) > 0$ and $f''(x) > 0$.

/2 (l) If $10 < x < 12$, then $f'(x) > 0$ and $f''(x) < 0$.



2. Compute the following derivatives. (Use derivative rules, rather than computing the limits.)

/2 (a) $f(x) = x^2 \Rightarrow f'(x) = 2x$

/2 (b) $f(x) = \frac{1}{x^2} \Rightarrow f'(x) = -2x^{-3}$

/2 (c) $f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$

/2 (d) $f(x) = \frac{1}{\sqrt{x}} \Rightarrow f'(x) = -\frac{1}{2}x^{-3/2}$

/2 (e) $f(x) = x^{3/4} \Rightarrow f'(x) = \frac{3}{4}x^{-1/4}$

/2 (f) $f(x) = \cos(x) \Rightarrow f'(x) = -\sin(x)$

/2 (g) $f(x) = \cos(7) \Rightarrow f'(x) = 0$

/2 (h) $f(x) = x^{-8} \Rightarrow f'(x) = -8x^{-9}$

/2 (i) $f(x) = x^2 + 7 \Rightarrow f'(x) = 2x$

/2 (j) $f(x) = 7x^2 \Rightarrow f'(x) = 14x$

/2 (k) $f(x) = e^x \Rightarrow f'(x) = e^x$

/2 (l) $f(x) = 5^x \Rightarrow f'(x) = 5^x \ln(5)$

/2 (m) $f(x) = \tan(x) \Rightarrow f'(x) = \sec^2(x)$

/2 (n) $f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$

/2 (o) $f(x) = \arctan(x) \Rightarrow f'(x) = \frac{1}{1+x^2}$

3. Compute the following derivatives:

/5 (a) $\dfrac{d}{dx}x^3\sin(x) =$

$3x^2\sin(x) + x^3\cos(x)$

/5 (b) $\dfrac{d}{dt}\dfrac{3t}{2+7t^3} =$

$\dfrac{3(2+7t^3) - 3t(7(3t^2))}{(2+7t^3)^2}$

/5 (c) $\dfrac{d}{dx}\left(3x^4 + \cot(x)\right)^9 =$

$9\left(3x^4 + \cot(x)\right)^8\left(3(4x^3) - \csc^2(x)\right)$

/5 (d) $\dfrac{d}{dx}\arcsin\left(\sqrt{1+x^4}\right) =$

$\dfrac{1}{\sqrt{1-\left(\sqrt{1+x^4}\right)^2}}\dfrac{1}{2}(1+x^4)^{-1/2}(4x^3)$

/5 (e) $\dfrac{d}{dt}\dfrac{t\cos(t)}{1+4^t} =$

$\dfrac{(1\cos(t) + t(-\sin(t)))(1+4^t) - t\cos(t)(4^t\ln(4))}{(1+4^t)^2}$

/10 4. Find $\frac{dy}{dx}$ using implicit differentiation for

$$y^2 \sin(x^2) = x \sin(y^2)\,.$$

Differentiating both sides with respect to x yields

$$2y\frac{dy}{dx}\sin(x^2) + y^2\cos(x^2)2x = \sin(y^2) + x\cos(y^2)2y\frac{dy}{dx}\,.$$

Gathering terms with $\frac{dy}{dx}$ to one side yields

$$2y\frac{dy}{dx}\sin(x^2) - x\cos(y^2)2y\frac{dy}{dx} = \sin(y^2) - y^2\cos(x^2)2x\,.$$

Factoring out $\frac{dy}{dx}$ yields

$$\frac{dy}{dx}\left(2y\sin(x^2) - x\cos(y^2)2y\right) = \sin(y^2) - y^2\cos(x^2)2x\,.$$

Solving for $\frac{dy}{dx}$ then yields

$$\frac{dy}{dx} = \frac{\sin(y^2) - y^2\cos(x^2)2x}{2y\sin(x^2) - x\cos(y^2)2y}\,.$$

/5 5. (a) State the Squeeze Theorem using the template below.

If • $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and

• $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L\,,$

then $\lim_{x\to a} g(x) = L\,.$

/6 (b) Use the Squeeze Theorem to evaluate $\lim_{x\to 0} 2x\sin\left(\frac{1}{x}\right)$.

Set

$$f(x) = -2|x|\,,$$
$$g(x) = 2x\sin\left(\frac{1}{x}\right)\,, \quad \text{and}$$
$$h(x) = 2|x|\,.$$

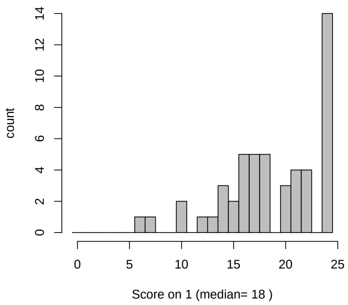
Since $-1 \le \sin(\cdot) \le 1$, we have $f(x) \le g(x) \le h(x)$ when x is near 0 (and for all $x \ne 0$), so the first assumption of the Squeeze Theorem holds with $a = 0$.

We can compute $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, so the second assumption of the Squeeze Theorem also holds with $a = 0$ and $L = 0$.

Thus the conclusion of the Squeeze Theorem holds and we can conclude

$$\lim_{x\to 0} 2x\sin\left(\frac{1}{x}\right) = \lim_{x\to a} g(x) = L = 0\,.$$

Scores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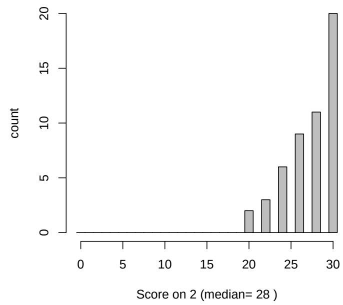
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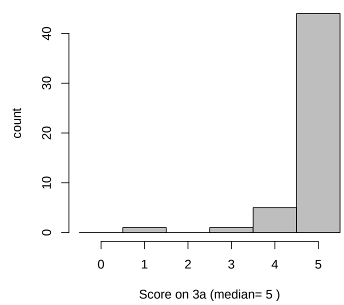
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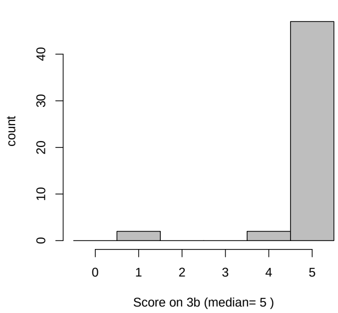
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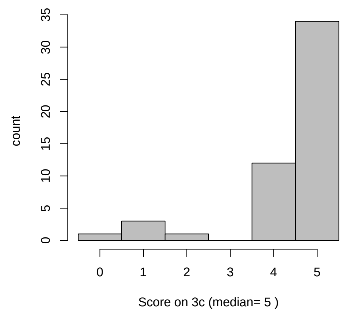
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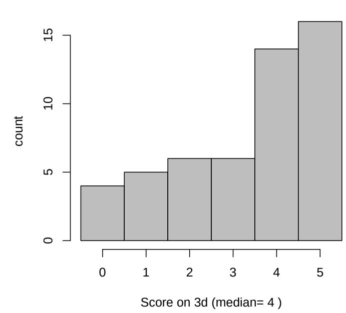
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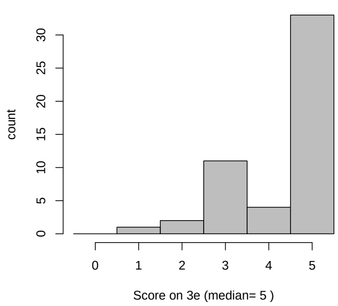
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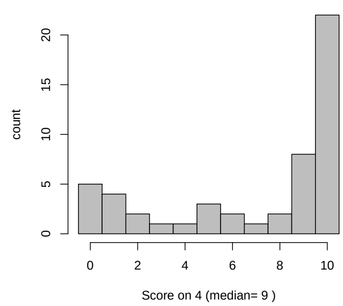
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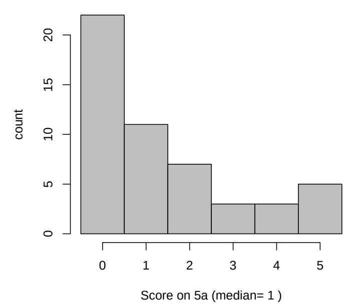
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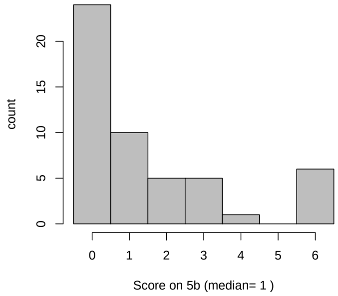
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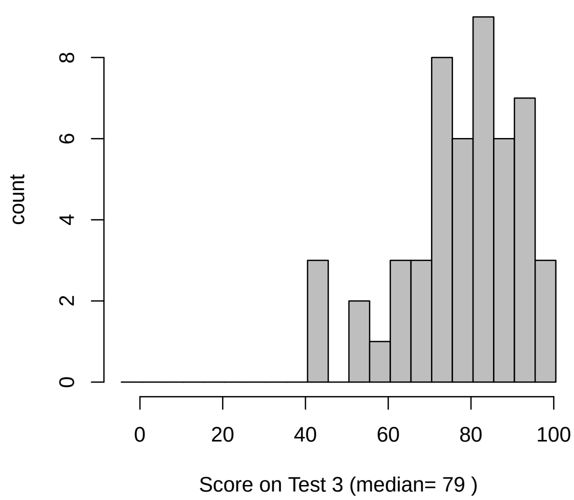
Score on 5b (median= 1)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Score on Test 3 (median= 79)