score	possible	page
	20	1
	20	2
	20	3
	40	4
	100	

Name: ____________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. A car is driven at 30 miles per hour for 20 minutes, stops for 10 minutes, and then is driven at 40 miles per hour for 30 minutes.

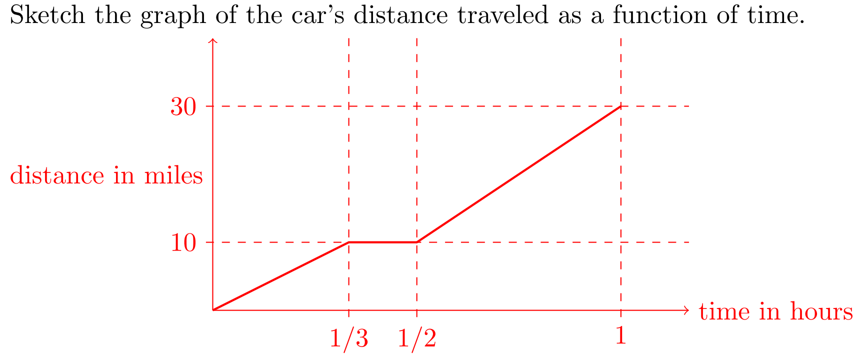
/10 (a) What is the average velocity of the car for this hour?

The car travels

$$30\frac{\text{mi}}{\text{hr}}(20\,\text{min})\frac{1\text{hr}}{60\,\text{min}} + 0 + 40\frac{\text{mi}}{\text{hr}}(30\,\text{min})\frac{1\text{hr}}{60\,\text{min}} = 10\text{mi} + 20\text{mi} = 30\text{mi}$$

in this hour, so its average velocity is $30\frac{\text{mi}}{\text{hr}}$.

/10 (b) Sketch the graph of the car's distance traveled as a function of time.



/5 2. (a) Write an equation for the line with slope 3 that passes through the point $(5, -7)$.

In point-slope form it is

$$y - (-7) = 3(x - 5).$$

(In slope-intercept form it is then $y = 3x - 22$.)

/5 (b) Write an equation for the line passing through the two points $(1, 2)$ and $(5, 7)$.

It has slope $m = \frac{7-2}{5-1} = \frac{5}{4}$ and so can be written in point-slope form as

$$y - 2 = \frac{5}{4}(x - 1).$$

(In slope-intercept form it is then $y = \frac{5}{4}x + \frac{3}{4}$.)

/10 (c) Find the x-coordinate of the point where these two lines intersect.

Using the slope-intercept forms of both equations and then setting them equal gives

$$\begin{aligned} 3x - 22 = \frac{5}{4}x + \frac{3}{4} \quad &\Leftrightarrow \\ \frac{7}{4}x = \frac{91}{4} \quad &\Leftrightarrow \\ x = \frac{91}{7} = 13\,. \end{aligned}$$

/10 3. Verify the identity $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$.

Multiplying both sides by $(1-\cos(\theta))(1+\cos(\theta))$ yields

$$\begin{aligned}(1+\cos(\theta)) + (1-\cos(\theta)) &= (1-\cos(\theta))(1+\cos(\theta))2\csc^2(\theta) \quad \Leftrightarrow \\ 2 &= (1-\cos^2(\theta))2\csc^2(\theta)\,.\end{aligned}$$

Remembering $1 = \sin^2(\theta) + \cos^2(\theta)$ and $\csc(\theta) = 1/\sin(\theta)$, we can simplify to

$$2 = \sin^2(\theta)2\frac{1}{\sin^2(\theta)} \quad \Leftrightarrow \quad 2 = 2\,,$$

so the original identity is verified.

/10 4. Solve the following equation for x: $\log_2(x+2) - 2\log_2(x) = 0$.

$$\Rightarrow \log_2\left(\frac{x+2}{x^2}\right) = 0 \qquad \Leftrightarrow \frac{x+2}{x^2} = 1 \qquad \Leftrightarrow x+2 = x^2$$
$$\Leftrightarrow x^2 - x - 2 = 0 \qquad \Leftrightarrow (x-2)(x+1) = 0 \qquad \Leftrightarrow x = 2 \text{ or } -1\,.$$

Since the domain of $\log_2$ is $(0,\infty)$ and we started with $\log_2(x)$, we know $x > 0$ and so can eliminate $x = -1$ as a solution.

5. Simplify and cancel so that you can plug in the given value without dividing by 0. Then plug in the value.

/10 (a) For $x = -3$, $\dfrac{x^2 + x - 6}{x + 3} =$

$$\frac{(x+3)(x-2)}{x+3} \quad = x - 2 \quad = -5\,.$$

(Fishy since we used both $x \neq -3$ and $x = -3$.)

/10 (b) For $x = 9$, $\dfrac{x - 9}{\sqrt{x} - 3} =$

$$\frac{x-9}{\sqrt{x}-3}\,\frac{\sqrt{x}+3}{\sqrt{x}+3} \quad = \frac{(x-9)(\sqrt{x}+3)}{x-9} \quad = \sqrt{x} + 3 \quad = 6\,.$$

(Fishy since we used both $x \neq 9$ and $x = 9$.)

/10 (c) For $h = 0$, $\dfrac{(x+h)^2 - x^2}{h} =$

$$\frac{(x^2 + 2xh + h^2) - x^2}{h} \quad = \frac{2xh + h^2}{h}$$

$$= \frac{h(2x+h)}{h} \quad = 2x + h \quad = 2x\,.$$

(Fishy since we used both $h \neq 0$ and $h = 0$.)

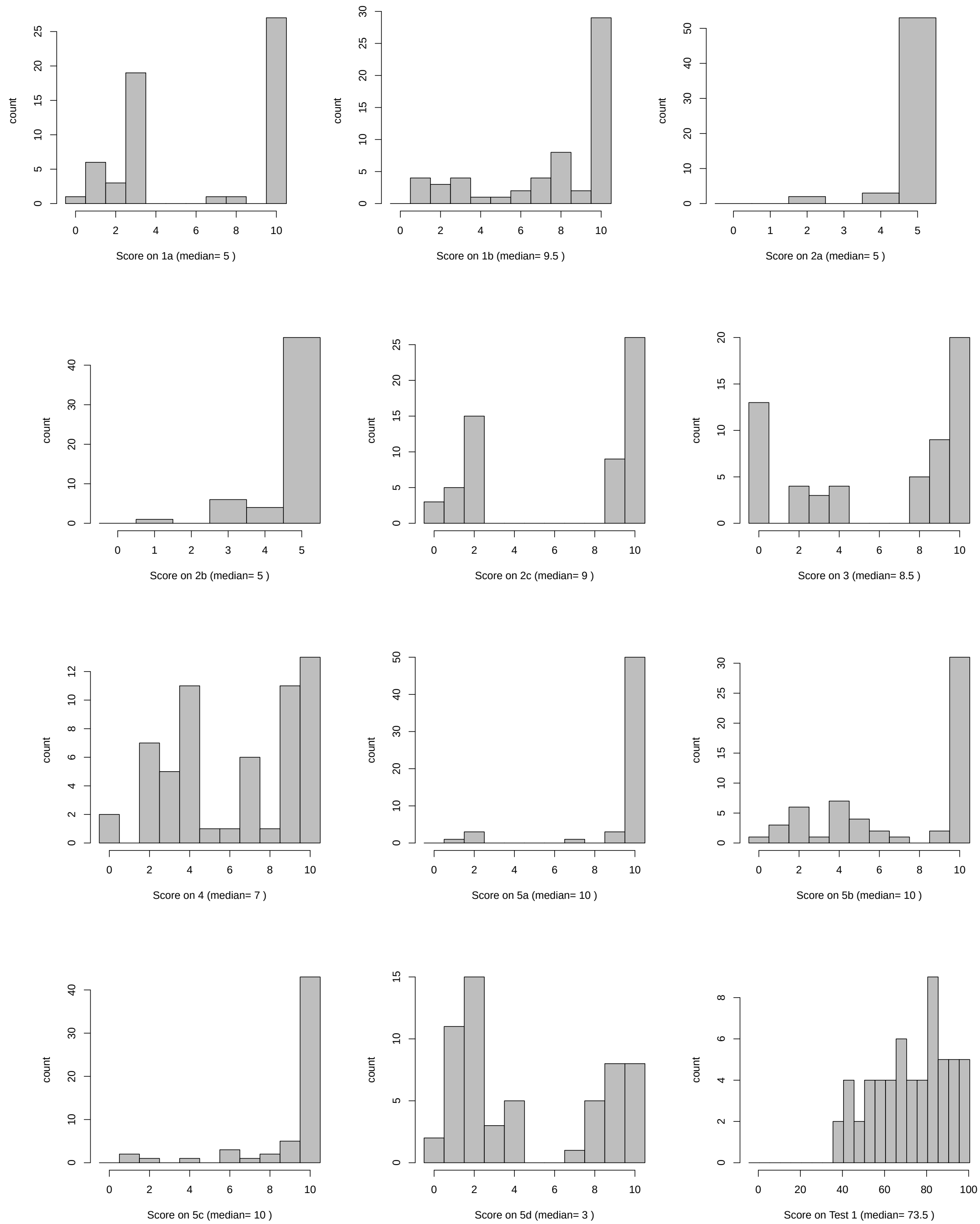
/10 (d) For $h = 0$, $\dfrac{(x+h)^{-1} - x^{-1}}{h} =$

$$\frac{\frac{1}{(x+h)} - \frac{1}{x}}{h} \quad = \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \quad = \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \frac{\frac{-h}{x(x+h)}}{h} \quad = \frac{-h}{hx(x+h)} \quad = \frac{-1}{x(x+h)} \quad = \frac{-1}{x^2}\,.$$

(Fishy since we used both $h \neq 0$ and $h = 0$.)

Scores


count
Score on 1a (median= 5)
Score on 1b (median= 9.5)
Score on 2a (median= 5)
Score on 2b (median= 5)
Score on 2c (median= 9)
Score on 3 (median= 8.5)
Score on 4 (median= 7)
Score on 5a (median= 10)
Score on 5b (median= 10)
Score on 5c (median= 10)
Score on 5d (median= 3)
Score on Test 1 (median= 73.5)