1. Suppose \( f \) and \( g \) are differentiable functions with the following properties:

\[
\begin{align*}
&f(0) = 2 & f(1) = 0 & f(2) = 1 \\
g(0) = 1 & g(1) = 2 & g(2) = 0 \\
\int_0^1 f(x) \, dx = \pi & \int_1^2 f(x) \, dx = \pi^3 & \int_2^3 f(x) \, dx = \pi^5 \\
\int_0 g(x) \, dx = \sqrt{2} & \int_1^2 g(x) \, dx = \sqrt{3} & \int_2 g(x) \, dx = \sqrt{5} \\
f'(0) = e & f'(1) = e^3 & f'(2) = e^5 \\
g'(0) = \sqrt{7} & g'(1) = \sqrt{11} & g'(2) = \sqrt{13}
\end{align*}
\]

Evaluate the following. If one cannot be evaluated with the given information, write “NOT ENOUGH INFORMATION.” You do not need to justify your answer or show your work.

\( /2 \) (a) \( \int_0^3 g(x) \, dx = \int_0^1 g(x) \, dx + \int_1^2 g(x) \, dx + \int_2^3 g(x) \, dx = \sqrt{2} + \sqrt{3} + \sqrt{5} \)

\( /2 \) (b) \( \int_3^2 f(x) \, dx = -\int_2^3 f(x) \, dx = -\pi^5 \)

\( /2 \) (c) \( \int_1^2 (5g(x) + f(x)) \, dx = 5\int_1^2 g(x) \, dx + \int_1^2 f(x) \, dx = 5\sqrt{3} + \pi^3 \)

\( /2 \) (d) \( \int_0^2 \frac{f(x)}{g(x)} \, dx \quad \text{Not enough information.} \)

\( /2 \) (e) \( \left( \frac{f}{g} \right)'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{(g(0))^2} = \frac{e \cdot 1 - 2\sqrt{7}}{1^2} \)

\( /2 \) (f) \( \int_0^9 f(x) \, dx - \int_2^9 f(x) \, dx = \int_0^1 f(x) \, dx + \int_1^2 f(x) \, dx = \pi + \pi^3 \)

\( /2 \) (g) \( \int_0^2 g'(r) \, dr = g(2) - g(0) = 0 - 1 \)

\( /2 \) (h) \( \int_7 g''(x) \, dx = 0 \)

\( /2 \) (i) \( \lim_{x \to 0} \frac{f(x) - 1}{g(x)} = \frac{2 - 1}{1} = 1 \)

\( /2 \) (j) \( \lim_{h \to 0} \frac{g(1 + h) - 2}{h} = g'(1) = \sqrt{11} \)
2. Use the Midpoint rule with \( n = 4 \) to approximate the integral. (Do \textbf{not} simplify.) Include a drawing of your subdivision of the interval and the midpoints used in the approximation.

\[
\int_{2}^{10} \sin(\sqrt{x})\,dx
\]

[Similar to 5.2#11] The interval \([a, b] = [2, 10]\) has length 8 and we are using 4 rectangles, so the width of each rectangle is \( \Delta x = 2 \). The base of the first rectangle is \([2, 4]\), which has midpoint \( x_1^* = 3 \). The second has base \([4, 6]\) and midpoint \( x_2^* = 5 \), the third has base \([6, 8]\) and midpoint \( x_3^* = 7 \), and the fourth has base \([8, 10]\) and midpoint \( x_4^* = 9 \). The area estimate is thus

\[
\sum_{i=1}^{4} f(x_i^*)\Delta x = \sin(\sqrt{3}) \cdot 2 + \sin(\sqrt{5}) \cdot 2 + \sin(\sqrt{7}) \cdot 2 + \sin(\sqrt{9}) \cdot 2.
\]

3. The velocity of a rabbit increased steadily during the first three seconds it tried to escape a fox. Its velocity at half second intervals is given in the table. Find good upper and lower estimates for the distance that it traveled during these three seconds. Do \textbf{not} simplify.

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>0</th>
<th>.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v ) (ft/s)</td>
<td>0</td>
<td>6.2</td>
<td>10.8</td>
<td>14.9</td>
<td>18.1</td>
<td>19.6</td>
<td>20.2</td>
</tr>
</tbody>
</table>

Since its velocity is increasing, using the left edge of each time interval will give a lower estimate and using the right edge will give an upper estimate. The lower estimate is

\[
\frac{1}{2} (0 + 6.2 + 10.8 + 14.9 + 18.1 + 19.6) \text{ ft}
\]

and the upper estimate is

\[
\frac{1}{2} (6.2 + 10.8 + 14.9 + 18.1 + 19.6 + 20.2) \text{ ft}.
\]

4. Sketch the function \( f(x) = 5 + \sqrt{4 - x^2} \) on the interval \([-2, 2]\).

Then evaluate the integral \( \int_{-2}^{2} f(x) \,dx \) by interpreting it in terms of areas.

\[
\int_{-2}^{2} f(x) \,dx = 20 + 2\pi.
\]
5. Evaluate the integrals. Do not simplify the result.

(a) \[\int_{-3}^{2} (x^2 - 7) \, dx = \left[ \frac{x^3}{3} - 7x \right]_{-3}^{2} = \frac{2^3}{3} - 7 \cdot 2 - \frac{(-3)^3}{3} - 7(-3).\]

(b) \[\int_{1}^{\pi} \frac{x^4 + 5}{x} \, dx = \left[ \frac{x^5}{5} + 5 \ln |x| \right]_{1}^{\pi} = \frac{\pi^5}{5} + 5 \ln(\pi) - \frac{1^5}{5} + 5 \ln(1).\]

(c) \[\int (x + 2)(5\sqrt{x} + 3) \, dx = \left[ 5x^{5/2} + 3x^{1/2} + 6x \right] = \frac{5}{2} x^{5/2} + \frac{3}{2} x^{3/2} + 6x + C.\]

(d) \[\int_{0}^{1/2} \frac{7}{\sqrt{1 - t^2}} \, dt = \left[ 7 \arcsin(t) \right]_{0}^{1/2} = 7 \arcsin(1/2) - 7 \arcsin(0).\]

(e) \[\int_{2}^{3} 10^{x^2 + 5} \, dx = \left[ \frac{10^y}{\ln(10)} \right]_{2}^{3} = 10^5 \frac{10^3}{\ln(10)} - 10^5 \frac{10^2}{\ln(10)}.\]
6. A cone-shaped drinking cup is made from a circular piece of paper of radius 7 in by cutting out a sector and joining the edges $CA$ and $CB$. Find the maximum capacity of such a cup.

[Similar to 4.5#29] A side view of the cone-shaped cup is the triangle

The volume of the cone is $V = \frac{1}{3}\pi r^2 h$. The pythagorean theorem gives the constraint $(7\text{ in})^2 = r^2 + h^2$. Solving for $r^2$ gives $r^2 = (7\text{ in})^2 - h^2$ and substituting into $V$ gives

$$V = \frac{1}{3}\pi((7\text{ in})^2 - h^2)h.$$ 

Differentiating with respect to $h$ gives

$$V' = \frac{1}{3}\pi((7\text{ in})^2 - 3h^2)$$

and differentiating again gives

$$V'' = \frac{1}{3}\pi(-6h).$$

Setting $V' = 0$ yields $h = \frac{7}{\sqrt{3}}$ in as the only positive critical number. Since $V'' < 0$ for $h > 0$, this critical number gives a maximum. Inserting into $V$ gives the maximum volume

$$V = \frac{1}{3}\pi\left(7^2 - \frac{7^2}{3}\right)\cdot\frac{7}{\sqrt{3}}\text{ in}^3 = \frac{2\cdot7^3\pi}{3\sqrt{3}}\text{ in}^3.$$
Scores

Score on 1 (median = 12)

Score on 2 (median = 10)

Score on 3 (median = 10)

Score on 4 (median = 5)

Score on 5a (median = 6)

Score on 5b (median = 4)

Score on 5c (median = 5)

Score on 5d (median = 3)

Score on 5e (median = 3)

Score on 6 (median = 3)

Score on Test 7 (median = 65)