1. Suppose $f$ and $g$ are differentiable functions with the following properties:

\[
\begin{array}{c}
\begin{align*}
 f(0) &= 2 & f(1) &= 0 & f(2) &= 1 \\
g(0) &= 1 & g(1) &= 2 & g(2) &= 0 \\
 f'(0) &= e & f'(1) &= e^3 & f'(2) &= e^5 \\
g'(0) &= \sqrt{7} & g'(1) &= \sqrt{11} & g'(2) &= \sqrt{13}
\end{align*}
\end{array}
\]

\[
\begin{array}{c}
\begin{align*}
 \int_0^1 f(x) \, dx &= \pi & \int_1^2 f(x) \, dx &= \pi^3 & \int_2^3 f(x) \, dx &= \pi^5 \\
 \int_0^1 g(x) \, dx &= \sqrt{2} & \int_1^2 g(x) \, dx &= \sqrt{3} & \int_2^3 g(x) \, dx &= \sqrt{5}
\end{align*}
\end{array}
\]

Evaluate the following. If one cannot be evaluated with the given information, write “NOT ENOUGH INFORMATION.” You do not need to justify your answer or show your work.

\( / 2 \)

(a) \[ \int_0^3 g(x) \, dx = \int_0^1 g(x) \, dx + \int_1^2 g(x) \, dx + \int_2^3 g(x) \, dx = \sqrt{2} + \sqrt{3} + \sqrt{5} \]

\( / 2 \)

(b) \[ \int_3^1 f(x) \, dx = -\int_2^3 f(x) \, dx = -\pi^5 \]

\( / 2 \)

(c) \[ \int_1^3 (5g(x) + f(x)) \, dx = 5 \int_1^2 g(x) \, dx + \int_1^2 f(x) \, dx = 5\sqrt{3} + \pi^3 \]

\( / 2 \)

(d) \[ \int_0^2 \frac{f(x)}{g(x)} \, dx \] Not enough information.

\( / 2 \)

(e) \[ \left( \frac{f}{g} \right)'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{(g(0))^2} = \frac{e \cdot 1 - 2\sqrt{7}}{1^2} \]

\( / 2 \)

(f) \[ \int_0^9 f(x) \, dx - \int_2^9 f(x) \, dx = \int_0^1 f(x) \, dx + \int_1^2 f(x) \, dx = \pi + \pi^3 \]

\( / 2 \)

(g) \[ \int_0^2 g'(r) \, dr = g(2) - g(0) = 0 - 1 \]

\( / 2 \)

(h) \[ \int_7^7 g''(x) \, dx = 0 \]

\( / 2 \)

(i) \[ \lim_{x \to 0} \frac{f(x) - 1}{g(x)} = \frac{2 - 1}{1} = 1 \]

\( / 2 \)

(j) \[ \lim_{h \to 0} \frac{g(1 + h) - 2}{h} = g'(1) = \sqrt{11} \]
2. Use the Midpoint rule with \( n = 4 \) to approximate the integral. (Do not simplify.) Include a drawing of your subdivision of the interval and the midpoints used in the approximation.

\[
\int_{1}^{9} \sin(\sqrt{x}) \, dx
\]

[Similar to 5.2#11] The interval \([a, b] = [1, 9]\) has length 8 and we are using 4 rectangles, so the width of each rectangle is \( \Delta x = 2 \). The base of the first rectangle is \([1, 3]\), which has midpoint \( x_1^* = 2 \). The second has base \([3, 5]\) and midpoint \( x_2^* = 4 \), the third has base \([5, 7]\) and midpoint \( x_3^* = 4 \), and the fourth has base \([7, 9]\) and midpoint \( x_4^* = 8 \). The area estimate is thus

\[
f(x_1^*) \Delta x + f(x_2^*) \Delta x + f(x_3^*) \Delta x + f(x_4^*) \Delta x = \sin(\sqrt{2}) \cdot 2 + \sin(\sqrt{4}) \cdot 2 + \sin(\sqrt{6}) \cdot 2 + \sin(\sqrt{8}) \cdot 2.
\]

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>0</th>
<th>.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v ) (ft/s)</td>
<td>0</td>
<td>5.2</td>
<td>10.8</td>
<td>14.9</td>
<td>18.1</td>
<td>19.6</td>
<td>20.2</td>
</tr>
</tbody>
</table>

Since its velocity is increasing, using the left edge of each time interval will give a lower estimate and using the right edge will give an upper estimate. The lower estimate is

\[
\frac{1}{2} (0 + 5.2 + 10.8 + 14.9 + 18.1 + 19.6) \text{ ft}
\]

and the upper estimate is

\[
\frac{1}{2} (5.2 + 10.8 + 14.9 + 18.1 + 19.6 + 20.2) \text{ ft}.
\]

4. Sketch the function \( f(x) = 3 + \sqrt{4 - x^2} \) on the interval \([-2, 2]\).

Then evaluate the integral \( \int_{-2}^{2} f(x) \, dx \) by interpreting it in terms of areas.

[Similar to 5.2#29–36] The integral is the area of a rectangle plus the area of half a disc. The rectangle has width 4 and height 3 and so area 12. The disc has radius 2 and so the half disc has area \( \frac{1}{2} \pi 2^2 = 2\pi \). Thus

\[
\int_{-2}^{2} f(x) \, dx = 12 + 2\pi.
\]
5. Evaluate the integrals. Do **not** simplify the result.

(a) \[ \int_{-3}^{2} (x^2 - 5) \, dx = \left[ \frac{x^3}{3} - 5x \right]_{-3}^{2} = \left( \frac{2^3}{3} - 5 \cdot 2 \right) - \left( \frac{(-3)^3}{3} - 5(-3) \right) . \]

(b) \[ \int_{1}^{\pi} \frac{x^4 + 3}{x} \, dx = \left[ \frac{x^5}{5} + 3 \ln(|x|) \right]_{1}^{\pi} = \left( \frac{\pi^5}{5} + 3 \ln(\pi) \right) - \left( \frac{1^5}{5} + 3 \ln(1) \right) . \]

(c) \[ \int (x + 2)(3\sqrt{x} + 5) \, dx = \left[ \frac{3x^{5/2}}{5/2} + 5x^{3/2} + 6x^{1/2} + 10x \right] = 3\frac{x^{5/2}}{5/2} + 5\frac{x^2}{2} + 6\frac{x^{3/2}}{3/2} + 10x + C . \]

(d) \[ \int_{0}^{1/2} \frac{5}{\sqrt{1 - t^2}} \, dt = \left[ 5 \arcsin(t) \right]_{0}^{1/2} = 5 \arcsin(1/2) - 5 \arcsin(0) . \]

(e) \[ \int_{2}^{3} 10^{x^2} \, dx = \left[ \frac{10^x}{\ln(10)} \right]_{2}^{3} = \frac{10^3}{\ln(10)} - \frac{10^2}{\ln(10)} . \]
6. A cone-shaped drinking cup is made from a circular piece of paper of radius 5 in by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.

[Similar to 4.5#29] A side view of the cone-shaped cup is the triangle

The volume of the cone is \( V = \frac{1}{3} \pi r^2 h \). The pythagorean theorem gives the constraint \((5 \text{ in})^2 = r^2 + h^2\). Solving for \( r^2 \) gives \( r^2 = (5 \text{ in})^2 - h^2 \) and substituting into \( V \) gives

\[
V = \frac{1}{3} \pi ((5 \text{ in})^2 - h^2) h.
\]

Differentiating with respect to \( h \) gives

\[
V' = \frac{1}{3} \pi((5 \text{ in})^2 - 3h^2)
\]

and differentiating again gives

\[
V'' = \frac{1}{3} \pi(-6h).
\]

Setting \( V' = 0 \) yields \( h = \frac{5}{\sqrt{3}} \text{ in} \) as the only positive critical number. Since \( V'' < 0 \) for \( h > 0 \), this critical number gives a maximum. Inserting into \( V \) gives the maximum volume

\[
V = \frac{1}{3} \pi \left(5^2 - \frac{5^2}{3}\right) \frac{5}{\sqrt{3}} \text{ in}^3 = \frac{2 \cdot 5^3 \pi}{3^2\sqrt{3}} \text{ in}^3.
\]
Scores

Score on 1 (median= 13 )

Score on 2 (median= 10 )

Score on 3 (median= 10 )

Score on 4 (median= 6 )

Score on 5a (median= 6 )

Score on 5b (median= 5.5 )

Score on 5c (median= 5 )

Score on 5d (median= 3 )

Score on 5e (median= 4 )

Score on 6 (median= 5.5 )

Score on Test 7 (median= 63.5 )