1. Determine whether each of the following statements is True or False. Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.

(a) True False If $x > 0$ then $\frac{\ln(x + 2)}{\ln(x)} = \ln(2)$. False. There are no simplifications for division of logarithms.

(b) True False If $x \neq 0$ then $\left(\frac{25}{4x^4}\right)\left(\frac{5}{x^3}\right)^{-3} = \frac{2x^5}{5}$. False. $\left(\frac{25}{4x^4}\right)\left(\frac{5}{x^3}\right)^{-3} = \left(\frac{25}{4x^4}\right)\left(\frac{x^3}{5}\right)^3 = \left(\frac{25x^9}{4x^4 \cdot 5^3}\right) = \frac{25x^9}{20} = \frac{x^5}{5} \neq \frac{2x^5}{5}$.

(c) True False If $x \neq 0$ then $x + x^{-1} = 0$. False. If $x = 1$ then $x + x^{-1} = 2$.

(d) True False If $\sin(t) \neq 0$ then $\sin(t) \cot(t) = \cos(t)$. True. $\cot(t) = \cos(t)/\sin(t)$.

(e) True False If $\lim_{x \to a} f(x) = f(a)$ then $f$ is continuous at $a$. True. That is the definition of continuous at a point.

(f) True False If $\lim_{x \to a} f(x) = f(a)$ then $f$ is differentiable at $a$. False. That is the definition of continuous at a point, which is necessary but not sufficient to make $f$ differentiable.

(g) True False The graph shows a jump discontinuity. True. The limits from each side exist but disagree, which is the definition of a jump discontinuity.

(h) True False If $f$ and $g$ are differentiable, then $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$. True. This is a basic differentiation formula, which the book calls the difference rule.

(i) True False A function can have two different horizontal asymptotes. True. It can have one asymptote as $x \to -\infty$ and another as $x \to \infty$.

(j) True False If $f(x) \leq h(x)$, $g(x) \leq f(x)$, $h(x) \leq 0$, $\lim_{x \to a} g(x) = K$, and $\lim_{x \to a} h(x) = K$, then $\lim_{x \to a} f(x) = K$. True. This is an awkwardly written version of the Squeeze Theorem. The assumption $h(x) \leq 0$ is not needed.
2. Compute the following limits. **Do not use L’Hôpital’s rule!**

\[
\begin{align*}
(a) \quad \lim_{x \to \pi^+} \csc(x) &= \\
&= \lim_{x \to \pi^+} \frac{1}{\sin(x)} = \lim_{t \to 0^+} \frac{1}{t} = -\infty. \text{ We used knowledge of the graph of } \sin(x), \text{ that } \sin(\pi) = 0 \text{ and } \\
&\quad \sin(x) < 0 \text{ for } x \text{ slightly bigger than } \pi.
\end{align*}
\]

\[
\begin{align*}
(b) \quad \lim_{x \to 4^+} \frac{x + 3}{x - 4} &= \\
&= \lim_{x \to 4^+} \frac{7}{x - 4} = \lim_{t \to 0^+} \frac{7}{t} = \infty.
\end{align*}
\]

\[
\begin{align*}
(c) \quad \lim_{x \to -\infty} \frac{5x^3 + 9x^5 + 2}{11x^5 - 13} &= \\
&\text{Multiplying the numerator and denominator by } x^{-5} \text{ yields} \\
&= \lim_{x \to -\infty} \frac{5x^{-2} + 9 + 2x^{-5}}{11 - 13x^{-5}} = \frac{9}{11}.
\end{align*}
\]

/10

3. Using the **definition of the derivative as a limit**, compute the derivative of \( f(x) = (3x - 1)^{-1} \).

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{1}{h} \left( \frac{1}{3(x + h) - 1} - \frac{1}{3x - 1} \right) \\
&= \lim_{h \to 0} \frac{-3h}{h(3x - 1)(3(x + h) - 1)} \\
&= \frac{-3}{(3x - 1)^2}.
\end{align*}
\]
4. Given that \( f(3) = 5 \) and \( f'(3) = 7 \), write an equation for the tangent line to the graph \( y = f(x) \) at \( x = 3 \).

Using point-slope form, the tangent line is \( y - f(a) = f'(a)(x - a) \). Plugging in the given values yields \( y - 5 = 7(x - 3) \).

5. Compute the following derivatives. (You can use derivative rules, rather than computing the limits.)

(a) \( f(x) = x^2 \Rightarrow f'(x) = 2x \)

(b) \( f(x) = \frac{1}{x^2} \Rightarrow f'(x) = -2x^{-3} \)

(c) \( f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2} \)

(d) \( f(x) = \frac{1}{\sqrt{x}} \Rightarrow f'(x) = -\frac{1}{2}x^{-3/2} \)

(e) \( f(x) = x^{3/4} \Rightarrow f'(x) = \frac{3}{4}x^{-1/4} \)

(f) \( f(x) = \sin(x) \Rightarrow f'(x) = \cos(x) \)

(g) \( f(x) = \sin(7) \Rightarrow f'(x) = 0 \)

(h) \( f(x) = x^{-7} \Rightarrow f'(x) = -7x^{-8} \)

(i) \( f(x) = x^2 + 5 \Rightarrow f'(x) = 2x \)

(j) \( f(x) = 5x^2 \Rightarrow f'(x) = 10x \)
6. (a) State the Intermediate Value Theorem using the template below.

If
• $f$ is continuous on $[a,b]$ and
• $f(a) < N < f(b)$ or $f(a) > N > f(b),$

then
there exists $c \in (a,b)$ such that $f(c) = N.$

(b) Use the Intermediate Value Theorem to show that the equation $\cos(x) = 2x^2$ has a solution.

Let $f(x) = \cos(x) - 2x^2,$ so we want to show a solution to $f(x) = 0$ exists. Since $\cos(x)$ and $x^2$ are both continuous, so is $f(x).$ Plugging in, we find

\[
\begin{align*}
f(0) &= 1 - 0 = 1 > 0 \quad \text{and} \\
f(\pi/2) &= 0 - 2(\pi/2)^2 < 0.
\end{align*}
\]

So, by the Intermediate Value Theorem, there must exist $0 < c < \pi/2$ such that $f(c) = 0.$

7. The graph of a function $f$ is given below. On the same axes, sketch the graph of $f'.$

![Graph of a function and its derivative](image)