The exam is cumulative. For material covered on previous tests, the previous guides and tests cover the types of questions that might be asked. The exam is 1.5 tests in length, so 6 (sides of) pages. Between a third and a half of the exam will be on new material, from Sections 10.2–10.5 and the supplement on Norms. Below are sample questions from those sections.

1. Determine whether each of the following statements is True or False.
   Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.
   Assume that the orders of the matrices are compatible so that they can be added or multiplied.

   (a) True False If $\mathbf{A}$ is square and you perform the QR-decomposition to get $\mathbf{A} = \mathbf{QR}$, then the diagonal entries of $\mathbf{R}$ are the eigenvalues of $\mathbf{A}$.

   (b) True False The purpose of the QR-algorithm is to compute the eigenvalues of a matrix $\mathbf{A}$.

   (c) True False If a set of vectors is *orthonormal*, then it is also *linearly independent*.

   (d) True False $\langle \mathbf{x} + \mathbf{z}, \mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{y} + \mathbf{z} \rangle$.

   (e) True False For any norm, $\|\mathbf{x} + \mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|$.

   (f) True False The *least-squares* solution to a system $\mathbf{Ax} = \mathbf{b}$ is the solution $\mathbf{x}$ with the smallest value for $\sum_{i=1}^{N} x_i^2$.

2. You did an experiment and collected the data

i	1	2	3	4
x_i	0	1	3	5
y_i	7	8	9	10

   . You decide to fit the data with a line $p(x)$. Set up a linear system to determine the coefficients of $p(x)$ and write the system as an augmented matrix. (Do not solve the system.)

3. For the vectors $\mathbf{x} = [\ 4 \quad 2 \quad -3\ ]^T$ and $\mathbf{y} = [\ 5 \quad -2 \quad -3\ ]^T$:

   (a) Find the projection of $\mathbf{x}$ onto $\mathbf{y}$.

   (b) Find the orthogonal complement of the projection of $\mathbf{x}$ onto $\mathbf{y}$.

4. Consider the system of equations $\left\{ \begin{array}{r} x + y = 2 \\ x + z = 3 \\ y + z = 4 \\ x + y + z = 5 \end{array} \right.$.

   (a) Write this system as a matrix equation.

   (b) Find the normal equations that would determine the least-squares solution to this system. Write them as a matrix equation. (Do not solve them.)

5. Use the Gram-Schmidt orthonormalization process to convert the columns of $\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix}$ to an orthonormal set.

6. Compute the QR-decomposition of the matrix $\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 0 \\ 0 & 2 & 3 \end{bmatrix}$.

7. Define what it means for a set of vectors $\{\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_n\}$ to be *orthonormal*.

8. Give an outline (pseudo-code) for the QR-algorithm. Explain how the results are interpreted.

9. For $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, compute

   (a) $\|\mathbf{x}\|_1$,

   (b) $\|\mathbf{x}\|_2$, and

   (c) $\|\mathbf{x}\|_\infty$.

10. For $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, compute

    (a) $\|\mathbf{A}\|_F$,

    (b) $\|\mathbf{A}\|_1$,

    (c) $\|\mathbf{A}\|_2$, and

    (d) $\|\mathbf{A}\|_\infty$.