The tests are cumulative. This guide gives some sample questions for Sections 6.1–6.5, Diagonalization, and Jordan Form. Doing these problems does not replace doing homework problems.

1. Define eigenvalue, eigenvector, trace, eigenspace, characteristic polynomial, algebraic multiplicity, geometric multiplicity, similar matrix, diagonalizable matrix, Jordan block, and Jordan normal form. [May be done as True/False questions checking that you know the definition.]

2. Determine whether each of the following statements is True or False.
   Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.
   Assume that the orders of the matrices are compatible so that they can be added or multiplied.

   (a) True False $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

   (b) True False If λ is an eigenvalue of $\mathbf{B}$ then $\lambda - 5$ is an eigenvalue of $\mathbf{B} - 5\mathbf{I}$.

   (c) True False If λ is an eigenvalue of $\mathbf{B}$ and $\mathbf{B}$ is invertible, then λ is an eigenvalue of $\mathbf{B}^{-1}$.

   (d) True False [More questions based on the properties in Section 6.4]

   (e) True False An eigenvalue's algebraic multiplicity is never more than its geometric multiplicity.

   (f) True False [More questions based on the Theorems in Section 6.5]

   (g) True False If $\mathbf{A}$ is invertible, then it is also diagonalizable.

   (h) True False [More questions based on diagonalization.]

   (i) True False If $\mathbf{A}$ is invertible, then it is also similar to a matrix in Jordan normal form.

   (j) True False [More questions based on Jordan normal form.]

3. For each matrix below, find all the eigenvalues.

   (a) [several 2×2 showing different possibilities, such as repeated eigenvalues, complex eigenvalues, etc.]

   (b) [a few bigger ones with structure that makes them easy to evaluate, such as being triangular.]

4. For each matrix below, the eigenvalues are given. For each eigenvalue, find a basis (of eigenvectors) for the eigenspace.

   (a) [one plain 2×2]

   (b) [one or two with something interesting, such as geometric multiplicity more than 1.]

5. The matrix $\mathbf{A}$ has eigenvalues

   - $\lambda_1 = **$ with algebraic multiplicity ** whose eigenspace has a basis $\{* * *\}$ and
   - $\lambda_2 = **$ with algebraic multiplicity ** whose eigenspace has a basis $\{* * *\}$.

   Find a matrix $\mathbf{P}$ that is invertible and a matrix $\mathbf{D}$ that is diagonal such that $\mathbf{A} = \mathbf{PDP}^{-1}$.

6. The matrix $\mathbf{A} =$ [some 3×3] has eigenvalues

   - $\lambda_1 = **$ with algebraic multiplicity ** whose eigenspace has a basis $\{* * *\}$ and
   - $\lambda_2 = **$ with algebraic multiplicity ** whose eigenspace has a basis $\{* * *\}$.

   Find a matrix $\mathbf{P}$ that is invertible and a matrix $\mathbf{J}$ that is in Jordan normal form such that $\mathbf{A} = \mathbf{PJP}^{-1}$.