The tests are cumulative. This guide gives some sample questions for Sections 2.7, 3.1–3.5, 5.1 and 5.2. Doing these problems does not replace doing homework problems.

- 1. Define inverse. Define minor. Define cofactor. [might be in True/False format]
- 2. State the relationship between the order (size) of  $\mathbf{A}$ , rank( $\mathbf{A}$ ), rank( $\mathbf{A}^{\mathbf{b}}$ ), the consistency of the system  $\mathbf{A}\mathbf{x} = \mathbf{b}$ , and the number of arbitrary unknowns in the solution of  $\mathbf{A}\mathbf{x} = \mathbf{b}$ .
- 3. Prove that  $\mathbf{A}^{-1}\mathbf{B}^{-1} = (\mathbf{B}\mathbf{A})^{-1}$ . [Theorems and properties from section 3.4. Some to prove, some as True/False.]
- 4. Given  $\mathbf{A}$  and  $\mathbf{b}$ , one method to solve  $\mathbf{A}\mathbf{x} = \mathbf{b}$  starts by forming the augmented matrix  $\mathbf{A}^{\mathbf{b}}$  and then using elementary row operations to transform it to row-reduced form. In each part below, the row-reduced form of  $\mathbf{A}^{\mathbf{b}}$  is given. Determine whether or not the system is consistent and find all solutions.

(a) 
$$\begin{bmatrix} 1 & 2 & 3 & | & 4 \\ 0 & 1 & 5 & | & 6 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

- (b) [several parts, mixed up: inconsistent, unique, 1 parameter, 2 parameter...]
- 5. Let  $\mathbf{A} = [3x3 \text{ with very easy numbers}].$ 
  - (a) Compute  $A^{-1}$ .
  - (b) Multiply  $\mathbf{A}\mathbf{A}^{-1}$  to check that it equals I. (Show your work.)
  - (c) Use  $\mathbf{A}^{-1}$  to solve  $\mathbf{A}\mathbf{x} = [3x1]$
- 6. Let  $\mathbf{A} = [3x3 \text{ with easy numbers}].$ 
  - (a) Compute the **LU** decomposition of **A**.
  - (b) Multiply **LU** to check that it equals **A**. (Show your work.)
  - (c) Use the **LU** decomposition to solve  $\mathbf{A}\mathbf{x} = [3x1]$ .
- 7. Compute the following determinants.
  - (a) |2x2 with variables| =
  - (b) |4x4 with some zeros| =