The tests are cumulative. This guide gives some sample questions for Sections 7.1–7.9, 10.1. Doing these problems does not replace doing homework problems.

1. (Expect a question taken from a previous test.)
2. Determine whether each of the following statements is True or False.
   Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.
   Assume that the orders of the matrices are compatible so that they can be added or multiplied.
   (a) True False If $\mathbf{A}$ is square, then $\mathbf{A}e^{\mathbf{A}} = e^{\mathbf{A}}\mathbf{A}$.
   (b) True False If $\mathbf{A}$ is square, then $e^{(\mathbf{A}^2)} = \left(e^{\mathbf{A}}\right)^2$.
   (c) True False If $\mathbf{A}(t)$ is square then $\frac{d}{dt}\left(\mathbf{A}(t)\right)^2 = \left(\frac{d}{dt}\mathbf{A}(t)\right)\mathbf{A}(t) + \mathbf{A}(t)\left(\frac{d}{dt}\mathbf{A}(t)\right)$.
   (d) True False If $\mathbf{A}(t)$ is square then $\frac{d}{dt}\left(\mathbf{A}(t)\right)^2 = 2\mathbf{A}(t)\left(\frac{d}{dt}\mathbf{A}(t)\right)$.
   (e) True False If $\mathbf{A}$ is square and $\mathbf{B}$ is invertible, then $\left(e^{\mathbf{A}t}\mathbf{B}\right)^{-1} = \mathbf{B}^{-1}e^{-\mathbf{A}t}$.
   (f) True False If $\mathbf{A}$ is square, then $e^{\mathbf{A}}e^{\mathbf{A}} = e^{2\mathbf{A}}$.
   (g) True False [others based on $e^{\mathbf{A}}$ properties in Section 7.8]
   (h) True False [others based on derivative properties in section 7.9]
3. You are trying to compute $e^{\mathbf{A}t}$ for a particular matrix $\mathbf{A}$ that has real entries but complex eigenvalues. You get complex-looking coefficients but want them to look real. Use Euler's relations to write the following coefficients without complex numbers:
   (a) $a_1 = \frac{e^{3t+\sqrt{15}ti} - e^{3t-\sqrt{15}ti}}{\sqrt{15}ti} =$
   (b) $a_0 = \frac{e^{3t+\sqrt{15}ti} + e^{3t-\sqrt{15}ti}}{4} =$
4. State the Cayley-Hamilton Theorem.
5. The characteristic polynomial of $\mathbf{A} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ is $p(\lambda) = -\lambda^3 + 4x^2 - 5x + 2$. Use the Cayley-Hamilton Theorem to compute $\mathbf{A}^{-1}$. (Do not compute $\mathbf{A}^{-1}$ by another method.)

6. The matrix $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 4$.

   (a) Compute $e^{\mathbf{A}t}$.

   (b) Compute $\frac{\mathrm{d}}{\mathrm{d}t} e^{\mathbf{A}t}$.

7. The matrix $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$ has eigenvalue $\lambda = 3$ with multiplicity 2. Compute $e^{\mathbf{A}}$.

8. For the vectors $\mathbf{x} = [\ 1 \quad 2 \quad -3\ ]^T$ and $\mathbf{y} = [\ 5 \quad -2 \quad -3\ ]^T$:

   (a) Compute $\langle \mathbf{x}, \mathbf{y} \rangle$.

   (b) Normalize $\mathbf{x}$.

9. You did an experiment and collected the data

i	1	2	3	4	5
x_i	0	.5	1.0	1.5	2.0
y_i	0	.19	.26	.29	.31

. You decide to use a polynomial $p(x)$ to interpolate the data, so it should have $p(x_i) = y_i$ for each i. Set up a linear system to determine the coefficients of $p(x)$ and write it as an augmented matrix. (Do not solve the system.)