Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student’s solutions. (You may ask me questions.)

1. The graph of a function $f$ is given in each part below. On the same axes, sketch the graph of $f'$. 
2. Let $f(x) = 1 + \sqrt{x}$.

(a) Using the definition of the derivative as a limit, compute $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{1 + \sqrt{x + h} - (1 + \sqrt{x})}{h} = \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} \frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} = \lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x + h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x + h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x + h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$ 

(b) Find the equation for the tangent line at $x = 4$.

The general form of the tangent line at $A$ is $y = f'(A)(x - A) + f(A)$ so we have

$$y = f'(4)(x - 4) + f(4) = \frac{1}{2\sqrt{4}}(x - 4) + (1 + \sqrt{4}) = \frac{1}{4}(x - 4) + 3.$$ 

(c) Graph $f(x)$ and the tangent line on the interval $[0, 8]$.

We can compute the $y$-intercept $f(0) = 1$ and the tangent line’s intercept $y = \frac{1}{4}(0 - 4) + 3 = 2$.

We already know they are tangent at $(4, 3)$. 

\[(4, 3)\]
3. Compute the following derivatives:

/10 (a) \( f(x) = 4 - x + x^3 + \frac{3}{x^2} - 2\sqrt{x} - 8x^{-7} + x^{3/4} + 3\cos(x) + \cot(x) - \sin(-7) \)

\[ \Rightarrow f'(x) = \\
0 - 1 + 3x^2 - 6x^{-3} - 2 \cdot \frac{x}{2\sqrt{x}} + 56x^{-8} + \frac{3}{4}x^{-1/4} - 3\sin(x) - \csc^2(x) - 0 \]

/10 (b) \[ D_x [(\cos(x) + x)(8\sin(x) + x^{-5})] = \\
(-\sin(x) + 1)(8\sin(x) + x^{-5}) + (\cos(x) + x)(8\cos(x) - 5x^{-6}) \]

/10 (c) \[ y = \frac{x^3 - 5x}{\sec(x) + x^2} \Rightarrow \frac{dy}{dx} = \\
\frac{(3x^2 - 5)(\sec(x) + x^2) - (x^3 - 5x)(\sec(x) \tan(x) + 2x)}{(\sec(x) + x^2)^2} \]
4. Compute the following limits. Show your work and/or explain your reasoning.

(a) \( \lim_{x \to 4^+} \frac{x + 3}{x + 4} = \)

\[
\lim_{x \to 4^+} \frac{-1}{x + 4} = \lim_{t \to 0^+} \frac{-1}{t} = -\infty.
\]

(b) \( \lim_{x \to -\infty} \frac{5x^3 + 7x^5 + 2}{11x^4 - 13} = \)

Multiplying the numerator and denominator by \( x^{-4} \) yields

\[
\lim_{x \to -\infty} \frac{5x^{-1} + 7x + 2x^{-4}}{11 - 13x^{-4}} = \lim_{x \to -\infty} \frac{7x}{11} = -\infty.
\]

5. (a) State the Squeeze Theorem. Identify its assumptions and its conclusions.

If (assumptions)

- \( f(x) \leq g(x) \leq h(x) \) when \( x \) is near \( a \) (except possibly at \( a \)) and
- \( \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L \),

then (conclusion)

\( \lim_{x \to a} g(x) = L \).

(b) Use the Squeeze Theorem to evaluate \( \lim_{x \to 0} \left( x^2 \cos \left( \frac{1}{x} \right) + 1 \right) \).

Set

\[
f(x) = -x^2 + 1, \quad g(x) = x^2 \cos \left( \frac{1}{x} \right) + 1, \quad \text{and} \quad h(x) = x^2 + 1.
\]

Since \( |\cos(\cdot)| \leq 1 \), we have \( f(x) \leq g(x) \leq h(x) \) when \( x \) is near 0 (and for all \( x \neq 0 \)). We can compute \( \lim_{x \to 0} f(x) = \lim_{x \to 0} h(x) = 1 \), so the assumptions of the Squeeze theorem are satisfied and we can conclude \( \lim_{x \to 0} g(x) = 1 \).