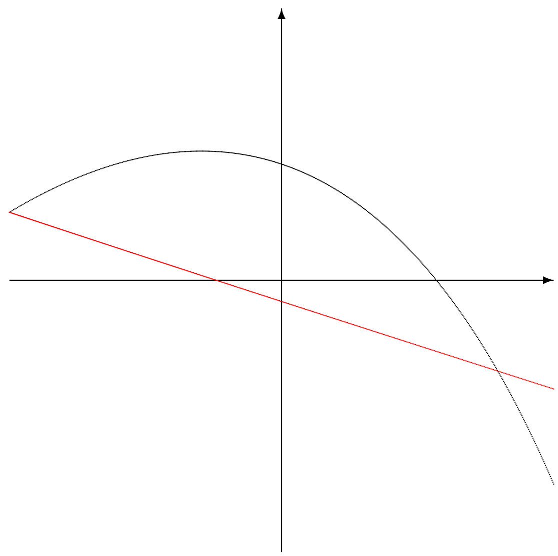
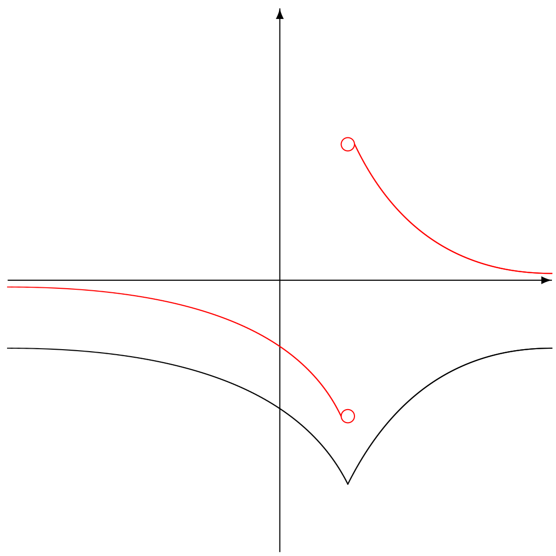
score	possible	page
	20	1
	30	2
	30	3
	20	4
	100	

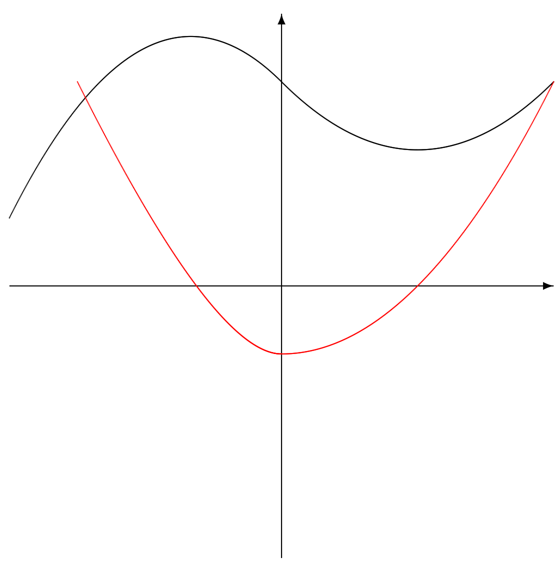
Name: ______________________________

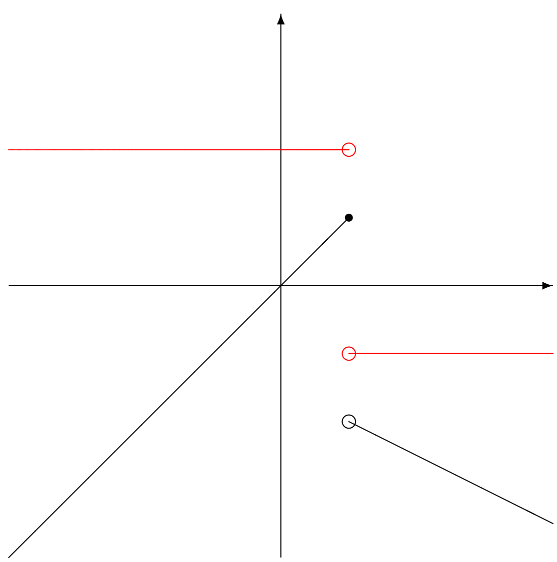
Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/20 1. The graph of a function f is given in each part below. On the same axes, sketch the graph of f'.









2. Let $f(x) = 1 + \sqrt{x}$.

/10 (a) Using **the definition of the derivative as a limit**, compute $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1 + \sqrt{x+h} - (1+\sqrt{x})}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$
$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}\,.$$

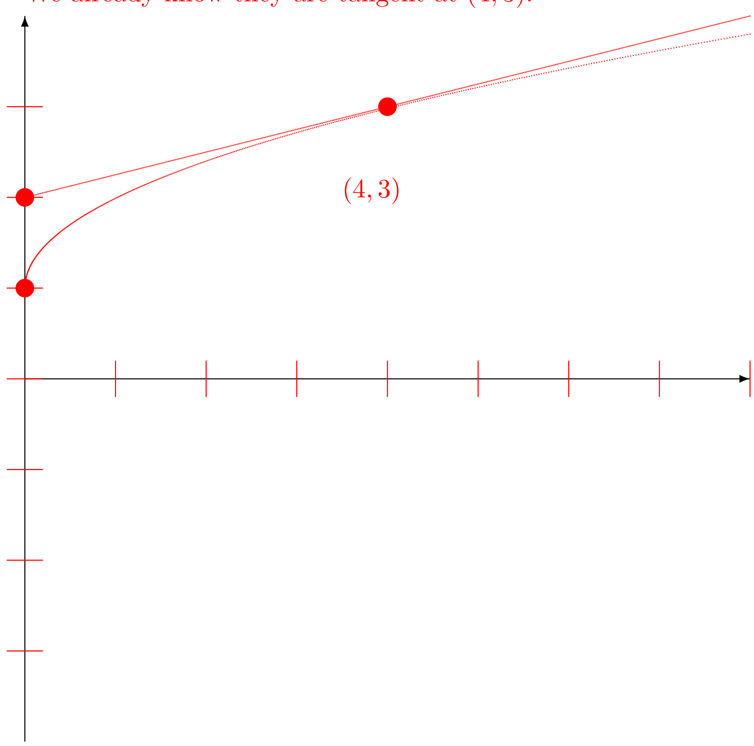
/10 (b) Find the equation for the tangent line at $x = 4$.

The general form of the tangent line at A is $y = f'(A)(x - A) + f(A)$ so we have

$$y = f'(4)(x-4) + f(4) = \frac{1}{2\sqrt{4}}(x-4) + (1+\sqrt{4}) = \frac{1}{4}(x-4) + 3\,.$$

/10 (c) Graph $f(x)$ and the tangent line on the interval $[0, 8]$.

We can compute the y-intercept $f(0) = 1$ and the tangent line's intercept $y = \frac{1}{4}(0-4)+3 = 2$. We already know they are tangent at $(4, 3)$.



3. Compute the following derivatives:

/10 (a) $f(x) = 4 - x + x^3 + \dfrac{3}{x^2} - 2\sqrt{x} - 8x^{-7} + x^{3/4} + 3\cos(x) + \cot(x) - \sin(-7)$
$\Rightarrow f'(x) =$

$$0 - 1 + 3x^2 - 6x^{-3} - \frac{2}{2\sqrt{x}} + 56x^{-8} + \frac{3}{4}x^{-1/4} - 3\sin(x) - \csc^2(x) - 0$$

/10 (b) $D_x\left[(\cos(x) + x)(8\sin(x) + x^{-5})\right] =$

$$(-\sin(x) + 1)(8\sin(x) + x^{-5}) + (\cos(x) + x)(8\cos(x) - 5x^{-6})$$

/10 (c) $y = \dfrac{x^3 - 5x}{\sec(x) + x^2} \Rightarrow \dfrac{dy}{dx} =$

$$\frac{(3x^2 - 5)(\sec(x) + x^2) - (x^3 - 5x)(\sec(x)\tan(x) + 2x)}{(\sec(x) + x^2)^2}$$

4. Compute the following limits. Show your work and/or explain your reasoning.

/5 (a) $\displaystyle\lim_{x\to -4^+} \frac{x+3}{x+4} =$

$$\lim_{x\to -4^+} \frac{-1}{x+4} = \lim_{t\to 0^+} \frac{-1}{t} = -\infty\,.$$

/5 (b) $\displaystyle\lim_{x\to -\infty} \frac{5x^3+7x^5+2}{11x^4-13} =$

Multiplying the numerator and denominator by x^{-4} yields

$$\lim_{x\to -\infty} \frac{5x^{-1}+7x+2x^{-4}}{11-13x^{-4}} = \lim_{x\to -\infty} \frac{7x}{11} = -\infty\,.$$

/5 5. (a) State the Squeeze Theorem. Identify its assumptions and its conclusions.

If (assumptions)

- $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and
- $$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L\,,$$

then (conclusion)

$$\lim_{x\to a} g(x) = L\,.$$

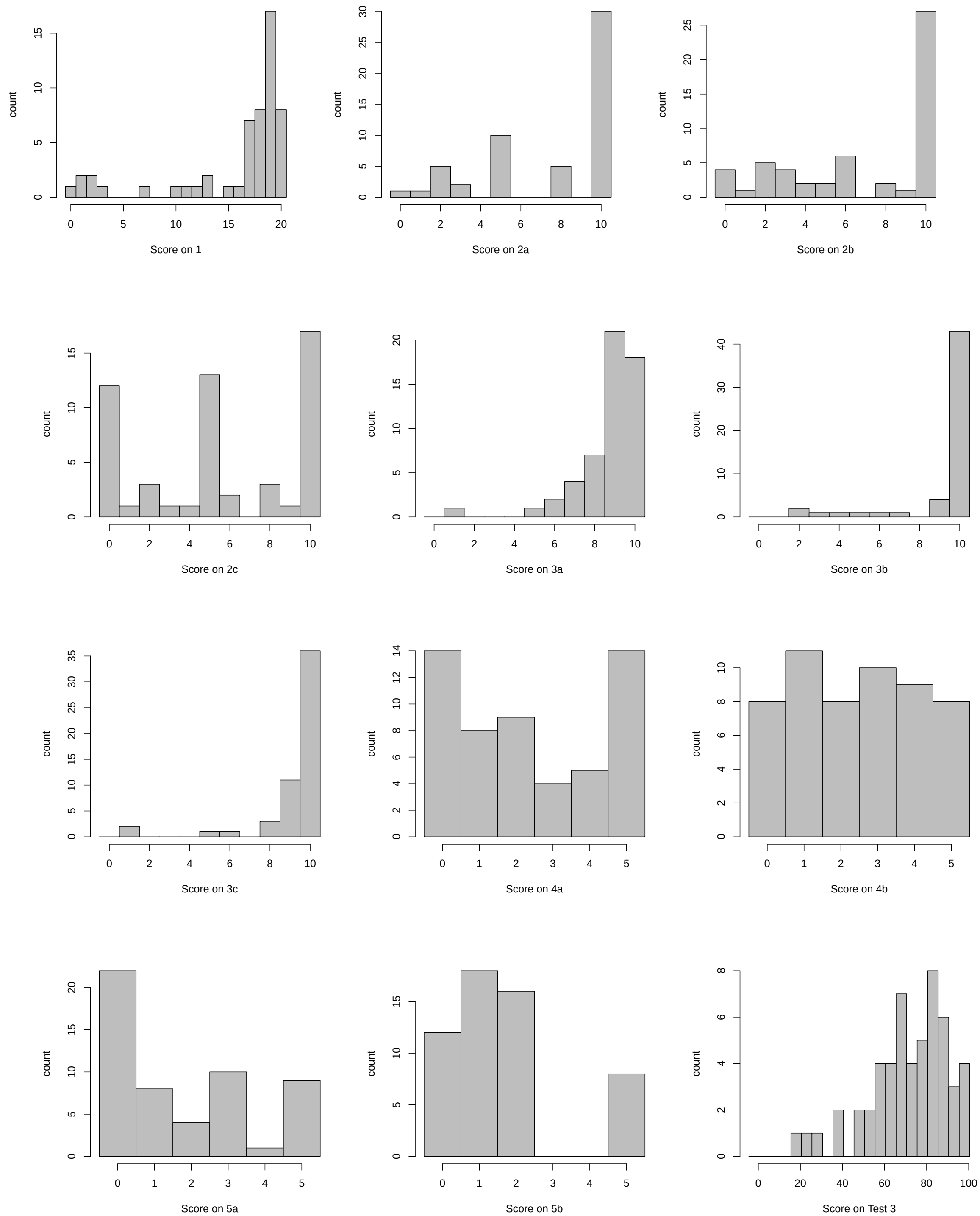
/5 (b) Use the Squeeze Theorem to evaluate $\displaystyle\lim_{x\to 0}\left(x^2\cos\left(\frac{1}{x}\right)+1\right)$.

Set

$$f(x) = -x^2+1\,,$$
$$g(x) = x^2\cos\left(\frac{1}{x}\right)+1\,, \quad \text{and}$$
$$h(x) = x^2+1\,.$$

Since $|\cos(\cdot)| \le 1$, we have $f(x) \le g(x) \le h(x)$ when x is near 0 (and for all $x \ne 0$). We can compute $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 1$, so the assumptions of the Squeeze theorem are satisfied and we can conclude $\lim_{x\to a} g(x) = 1$.

Scores


count
Score on 1
Score on 2a
Score on 2b
Score on 2c
Score on 3a
Score on 3b
Score on 3c
Score on 4a
Score on 4b
Score on 5a
Score on 5b
Score on Test 3