Work in groups of 3 or 4. Show your work. Acknowledge any help on these specific problems.

1. Use the formula

\[ \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{\infty} f(x_i) \Delta x \]

with \( \Delta x = (b - a)/n \) and \( x_i = a + i\Delta x \) to evaluate the integral

\[ \int_{0}^{1} (x^3 - 3x^2) \, dx . \]
2. Suppose $f$ and $g$ are differentiable functions with the following properties:

\[
\begin{align*}
    f(0) &= 2 & f(1) &= 0 & f(2) &= 1 \\
    g(0) &= 1 & g(1) &= 2 & g(2) &= 0 \\
    \int_0^1 f(x) dx &= \pi & \int_1^2 f(x) dx &= \pi^3 & \int_2^3 f(x) dx &= \pi^5 \\
    \int_0^1 g(x) dx &= \sqrt{2} & \int_1^2 g(x) dx &= \sqrt{3} & \int_2^3 g(x) dx &= \sqrt{5} \\
    f'(0) &= e & f'(1) &= e^3 & f'(2) &= e^5 \\
    g'(0) &= \sqrt{7} & g'(1) &= \sqrt{11} & g'(2) &= \sqrt{13}
\end{align*}
\]

Evaluate the following. If one cannot be evaluated with the given information, write “NOT ENOUGH INFORMATION.”

(a) $\int_0^1 f(r) dr$

(b) $\int_0^3 f(x) dx$

(c) $\int_3^2 g(x) dx$

(d) $\int_1^2 (5f(x) + g(x)) dx$

(e) $\int_0^1 f(x) g(x) dx$

(f) $\int_0^{14} f(x) dx - \int_2^{14} f(x) dx$

(g) $\int_0^2 f'(r) dr$

(h) $\int_6^6 f''(x) dx$

(i) $\lim_{x \to 1} \frac{f(x)}{g(x) - 2}$

(j) $\lim_{h \to 0} \frac{f(2 + h) - 1}{h}$
3. (a) Evaluate the integral $\int_{-1}^{2} |x| \, dx$ by interpreting it in terms of area.

(b) Evaluate the integral $\int_{-1}^{2} x^3 \, dx$ and interpret it as a difference of areas. Illustrate with a sketch.
4. Evaluate the integrals:

/5  (a) \( \int_{-2}^{3} (x^2 - 3) \, dx = \)

/5  (b) \( \int_{1}^{4} \left( \frac{4 + 6u}{\sqrt{u}} \right) \, du = \)

/5  (c) \( \int_{1}^{2} \left( \frac{x}{2} - \frac{2}{x} \right) \, dx = \)

/5  (d) \( \int_{1}^{e} \frac{x^2 + x + 1}{x} \, dx = \)

/5  (e) \( \int \frac{\sin(x)}{1 - \sin^2(x)} \, dx = \)

/5  (f) \( \int \tan(7) \, dx = \)