score	possible	problem
	25	1
	25	2
	10	3
	15	4
	25	5
	100	

Name: _____
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Work in groups of 3 or 4. Show your work. Aknowledge any help on these specific problems.

/25 1. A company wishes to manufacture a box with a volume of 6 m^3 that is open on top and has a square base. The material for the bottom of the box costs \$3 per m^2, while the material for the sides costs \$2 per m^2. Find the dimensions of the box that will lead to minimum total cost. What is the minimum total cost?

2. For integers $m \leq n$ and some numbers $\{a_i\}$, sigma notation is defined by

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_n \,.$$

The following are proposed properties of sigma notation. For each of them, either

- explain/ prove/ show that it is (always) true or
- give an example showing that it is false.

/3 (a) $\displaystyle\sum_{i=m}^{n} a_i = \sum_{j=m}^{n} a_j$

/3 (b) $\displaystyle\sum_{i=m}^{n} a_i = \sum_{i=0}^{n-m} a_{i-m}$

/3 (c) $\displaystyle\sum_{i=m}^{n} (3a_i) = 3\left(\sum_{i=m}^{n} a_i\right)$

/3 (d) $\displaystyle\sum_{i=m}^{n} a_{3i} = \sum_{i=3m}^{3n} a_i$

/3 (e) $\displaystyle\sum_{i=m}^{n} (a_i + b_i) = \left(\sum_{i=m}^{n} a_i\right) + \left(\sum_{i=m}^{n} b_i\right)$

/3 (f) $\displaystyle\sum_{i=m}^{n} (a_i b_i) = \left(\sum_{i=m}^{n} a_i\right)\left(\sum_{i=m}^{n} b_i\right)$

/3 (g) $\displaystyle\sum_{i=m}^{n} |a_i| = \left|\sum_{i=m}^{n} a_i\right|$

/4 (h) $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

/10 3. 4-methylcyclohexanemethanol (MCHM) leaked from a tank at a rate of $r(t)$ liters (l) per hour (h). The rate decreased as time passed and the values of the rate at two-hour intervals are shown in the table. Find lower and upper estimates for the total amount of MCHM that leaked out.

$t(h)$	0	2	4	6	8	10
$r(t)$ (l/h)	8.7	7.6	6.8	6.2	5.7	5.3

/15 4. A brick falls from the top of a tall building 450 m above the ground.

(a) Find the distance of the brick above ground level at time t.

(b) How long does it take the brick to hit the ground?

(c) With what velocity does it hit the ground?

/25 5. For the function $f(x) = \frac{1}{2}x - \sin(x)$ on the interval $0 < x < 3\pi$:

(a) Find any asymptotes.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find the local maximum and minimum values of f.

(d) Find the intervals of concavity and the inflection points.

(e) Use the information above to sketch the graph.