score	possible	page
	20	1
	25	2
	25	3
	30	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/10 1. Solve the equation and describe all solutions that lie in the interval $[0, 2\pi)$:

$$2\sin^2(\theta) = 5\sin(\theta) + 3\,.$$

$$\Leftrightarrow 0 = 2\sin^2(\theta) - 5\sin(\theta) - 3$$
$$\Leftrightarrow 0 = (2\sin(\theta) + 1)(\sin(\theta) - 3)$$
$$\Leftrightarrow \sin(\theta) = -1/2 \quad \text{or} \quad \sin(\theta) = 3\,.$$

The first equation has solutions $\theta = 7\pi/6$ and $\theta = 11\pi/6$. The second equation has no solutions.

/10 2. Use the properties of logarithms to rewrite the following expressions:

- Write as a single logarithm:

$$\log_2(x^3 - 4) + 2\log_2(x + 2) - 4\log_2(3x + 2)$$

- Write so that the result does not contain any powers, products, or quotients:

$$\log_3\left(\frac{xy}{z^2\sqrt[3]{w}}\right)$$

- $$\log_2\left(\frac{(x^3 - 4)(x + 2)^2}{(3x + 2)^4}\right)$$

- $$\log_3(x) + \log_3(y) - 2\log_3(z) - \frac{1}{3}\log_3(w)$$

/25 3. Consider the rational function

$$f(x) = \frac{x^2 + 4x + 3}{x^2 - 1}\,.$$

(a) Express the domain of f in interval notation.

(b) Find the x and y intercepts of f.

(c) Find all vertical and horizontal asymptotes.

(d) Identify any holes.

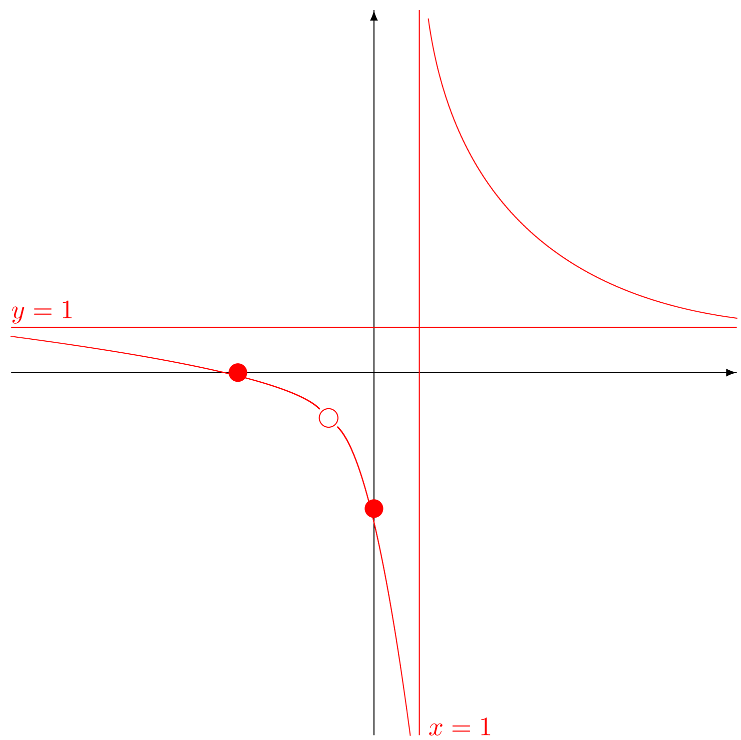
(e) Sketch a detailed graph of f.

$f(x) = \frac{(x+1)(x+3)}{(x+1)(x-1)} = \frac{x+3}{x-1}$ except that there is a hole at $x = -1$. Since we also divide by 0 at $x = 1$, the domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

$f(0) = \frac{0+3}{0-1} = -3$ so the y-intercept is at $(0, -3)$. Setting $0 = \frac{x+3}{x-1}$ yields $x = -3$, so the x-intercept is at $(-3, 0)$.

Since $(x - 1)$ remains in the denominator, $x = 1$ is a vertical asymptote. Horizontal asymptotes are determined by the highest powers in the numerator and denominator, so we have $\frac{x+3}{x-1} \to \frac{x}{x} = 1$ and $y = 1$ is a horizontal asymptote.

As noted above, $x = -1$ gives a hole. The y-value is $\frac{-1+3}{-1-1} = -1$.



/15 4. Let

$$g(x) = \begin{cases} \dfrac{2x^2 - x - 15}{x - 3} & \text{if } x \neq 3 \\ kx - 1 & \text{if } x = 3 \end{cases}.$$

Determine the value of k that will make the function g continous, or explain why no value of k will work.

For $x \neq 3$ we can reduce to

$$\frac{2x^2 - x - 15}{x - 3} = \frac{(x-3)(2x+5)}{x-3} = 2x + 5\,.$$

From this we know that g is continous when $x \neq 3$ and $\lim_{x \to 3} = 2(3) + 5 = 11$. To make g also continuous at $x = 3$, we need $g(3) = k(3) - 1 = 11$, which means $k = 4$.

/10 5. Use the Intermediate Value Theorem to show that the equation $2^{-x} = x$ has a solution.

Let $f(x) = x - 2^{-x}$, so we want to show a solution to $f(x) = 0$ exists. Since x and 2^{-x} are both continuous, so is $f(x)$. Plugging in, we find

$$f(0) = 0 - 2^0 = -1 < 0 \quad \text{and}$$
$$f(1) = 1 - 2^{-1} = 1/2 > 0\,.$$

So, by the Intermediate Value Theorem, there must exist $0 < c < 1$ such that $f(c) = 0$.

6. Compute the following limits. If you use the Squeeze Theorem, then indicate the two functions that you are using to squeeze.

/10 (a) For $f(x) = (2x+1)^{-1}$, compute $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} =$

$$
\begin{aligned}
&= \lim_{h\to 0} \frac{(2(x+h)+1)^{-1} - (2x+1)^{-1}}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{(2(x+h)+1)} - \frac{1}{(2x+1)}}{h} \\
&= \lim_{h\to 0} \frac{\frac{(2x+1)-(2(x+h)+1)}{(2x+1)(2(x+h)+1)}}{h} \\
&= \lim_{h\to 0} \frac{-2h}{h(2x+1)(2(x+h)+1)} \\
&= \lim_{h\to 0} \frac{-2}{(2x+1)(2(x+h)+1)} \\
&= \frac{-2}{(2x+1)(2(x+0)+1)} = \frac{-2}{(2x+1)^2}\,.
\end{aligned}
$$

/10 (b) $\lim_{x\to 0} 4x\cos(3/x) =$

Since the range of cos is $[-1, 1]$, we have

$$
\begin{aligned}
&|4x\cos(3/x)| \le 4|x| \quad \text{and so} \\
&\quad -4|x| \le 4x\cos(3/x) \le 4|x|\,.
\end{aligned}
$$

We have $\lim_{x\to 0} -4|x| = 0 = \lim_{x\to 0} 4|x|$. So by the squeeze theorem, using the two functions $-4|x|$ and $4|x|$ to squeeze, we know $\lim_{x\to 0} 4x\cos(3/x) = 0$.

/10 (c) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3} =$

$$
\begin{aligned}
&= \lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}\,\frac{\sqrt{x}+3}{\sqrt{x}+3} \\
&= \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{x-9} \\
&= \lim_{x\to 9} (\sqrt{x}+3) \\
&= \sqrt{9} + 3 = 6\,.
\end{aligned}
$$