score	possible	page
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	100	

Name: ________________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/10 1. Verify the identity $\dfrac{1}{1-\cos(\theta)} + \dfrac{1}{1+\cos(\theta)} = 2\csc^2(\theta)$.

Putting onto a common denominator, we obtain

$$\frac{1+\cos(\theta)+1-\cos(\theta)}{(1-\cos(\theta))(1+\cos(\theta))} = 2\csc^2(\theta)\,.$$

Canceling in the numerator and expanding in the denominator yields

$$\frac{2}{1-\cos^2(\theta)} = 2\csc^2(\theta)\,.$$

Recalling $1 = \sin^2(\theta)+\cos^2(\theta)$ and $\csc(\theta) = \frac{1}{\sin(\theta)}$, we get

$$\frac{2}{\sin^2(\theta)} = 2\left(\frac{1}{\sin(\theta)}\right)^2\,,$$

so the identity is verified.

/10 2. Solve the following equation for x: $\log_3(x-4)+\log_3(x+4) = 2$.

$$\begin{aligned}
&\Leftrightarrow \log_3\left((x-4)(x+4)\right) = 2\\
&\Leftrightarrow (x-4)(x+4) = 3^2\\
&\Leftrightarrow x^2-16 = 9\\
&\Leftrightarrow x^2 = 25\\
&\Leftrightarrow x = \pm 5
\end{aligned}$$

(Since the domain of $\log_3$ is $(0,\infty)$ and we started with $\log_3(x-4)$, we know $x > 4$ and so could eliminate $x = -5$ as a solution.)

3. The function $f(x) = -7 + \sqrt[7]{4x-5}$ is one-to-one on its domain.

/10 (a) Find a formula for its inverse, $f^{-1}(x)$.

$$\begin{aligned} y &= -7 + \sqrt[7]{4x-5} \\ \Leftrightarrow y + 7 &= (4x-5)^{1/7} \\ \Leftrightarrow (y+7)^7 &= 4x - 5 \\ \Leftrightarrow \frac{(y+7)^7+5}{4} &= x \end{aligned}$$

so $f^{-1}(x) = \frac{(x+7)^7+5}{4}$.

/10 (b) Verify your formula is correct by computing and simplifying $f \circ f^{-1}(x)$.

$$\begin{aligned} f \circ f^{-1}(x) &= -7 + \sqrt[7]{4\frac{(x+7)^7+5}{4} - 5} \\ &= -7 + \sqrt[7]{(x+7)^7 + 5 - 5} \\ &= -7 + \sqrt[7]{(x+7)^7} \\ &= -7 + (x+7) \\ &= x \end{aligned}$$

/30 4. Consider the rational function

$$f(x) = \frac{3x^2 - 3x}{x^2 - 5x + 4}\,.$$

(a) Express the domain of f in interval notation.

(b) Find the x and y intercepts of f.

(c) Find all vertical and horizontal asymptotes.

(d) Identify any holes.

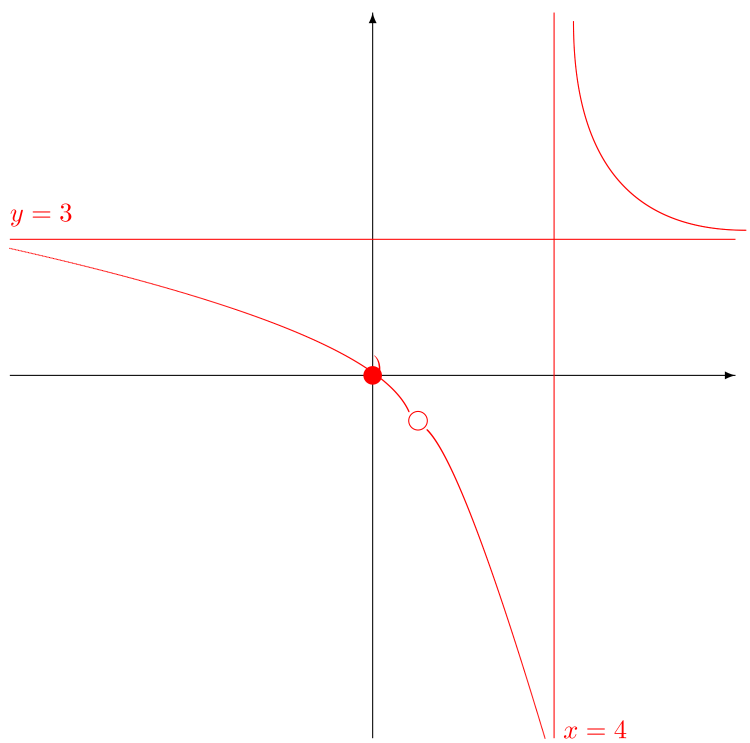
(e) Sketch a detailed graph of f.

$f(x) = \frac{3x(x-1)}{(x-4)(x-1)} = \frac{3x}{x-4}$ except that there is a hole at $x = 1$. Since we also divide by 0 at $x = 4$, the domain is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$.

$f(0) = \frac{3(0)}{0-4} = 0$ so the y-intercept is at $(0, 0)$. Setting $0 = \frac{3x}{x-4}$ yields $x = 0$, so the x-intercept is also at $(0, 0)$.

Since $(x - 4)$ remains in the denominator, $x = 4$ is a vertical asymptote. Horizontal asymptotes are determined by the highest powers in the numerator and denominator, so we have $\frac{3x}{x-4} \to \frac{3x}{x} = 3$ and $y = 3$ is a horizontal asymptote.

As noted above, $x = 1$ gives a hole. The y-value is $\frac{3(1)}{1-4} = -1$.



5. Simplify and cancel so that you can plug in the given value without dividing by 0.

/10 (a) For $x = 2$, $\dfrac{x^2 + x - 6}{x - 2} =$

$$
\begin{aligned}
&= \frac{(x+3)(x-2)}{x-2} \\
&= (x+3) \\
&= 5
\end{aligned}
$$

(Fishy since we used both $x \neq 2$ and $x = 2$.)

/10 (b) For $h = 0$, $\dfrac{(x+h)^2 - x^2}{h} =$

$$
\begin{aligned}
&= \frac{x^2 + 2xh + h^2 - x^2}{h} \\
&= \frac{2xh + h^2}{h} \\
&= \frac{h(2x+h)}{h} \\
&= 2x + h \\
&= 2x
\end{aligned}
$$

(Fishy since we used both $h \neq 0$ and $h = 0$.)

/10 (c) For $h = 0$, $\dfrac{(x+h)^{-1} - x^{-1}}{h} =$

$$
\begin{aligned}
&= \frac{\frac{1}{(x+h)} - \frac{1}{x}}{h} \\
&= \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\
&= \frac{\frac{x-(x+h)}{x(x+h)}}{h} \\
&= \frac{\frac{-h}{x(x+h)}}{h} \\
&= \frac{-h}{hx(x+h)} \\
&= \frac{-1}{x(x+h)} \\
&= \frac{-1}{x^2}
\end{aligned}
$$

(Fishy since we used both $h \neq 0$ and $h = 0$.)