Work in groups of 3 or 4. Show your work. Acknowledge any help on these specific problems.

1. Use the formula
\[ \int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{\infty} f(x_i) \Delta x \]
with \( \Delta x = (b - a)/n \) and \( x_i = a + i \Delta x \) to evaluate the integral
\[ \int_0^1 (x^3 - 3x^2)dx \].
2. Suppose $f$ and $g$ are differentiable functions with the following properties:

\[ f(0) = 2 \quad f(1) = 0 \quad f(2) = 1 \]
\[ g(0) = 1 \quad g(1) = 2 \quad g(2) = 0 \]
\[ \int_0^1 f(x)dx = \pi \quad \int_1^2 f(x)dx = \pi^3 \quad \int_2^3 f(x)dx = \pi^5 \]
\[ \int_0^1 g(x)dx = \sqrt{2} \quad \int_1^2 g(x)dx = \sqrt{3} \quad \int_2^3 g(x)dx = \sqrt{5} \]
\[ f'(0) = e \quad f'(1) = e^3 \quad f'(2) = e^5 \]
\[ g'(0) = \sqrt{7} \quad g'(1) = \sqrt{11} \quad g'(2) = \sqrt{13} \]

Evaluate the following. If one cannot be evaluated with the given information, write “NOT ENOUGH INFORMATION.”

(a) \( \int_0^1 f(r)dr \)

(b) \( \int_0^2 f(x)dx \)

(c) \( \int_3^2 g(x)dx \)

(d) \( \int_1^2 (5f(x) + g(x))dx \)

(e) \( \int_0^1 f(x)g(x)dx \)

(f) \( \int_0^{14} f(x)dx - \int_2^{14} f(x)dx \)

(g) \( \int_0^2 f'(r) \, dr \)

(h) \( \int_6^6 f''(x) \, dx \)

(i) \( \lim_{x \to 1} \frac{f(x)}{g(x) - 2} \)

(j) \( \lim_{h \to 0} \frac{f(2 + h) - 1}{h} \)
3. (a) Evaluate the integral \( \int_{-1}^{2} |x| \, dx \) by interpreting it in terms of area.

(b) If \( f(0) = 0 \) and \( 3 \leq f'(x) \leq 5 \), what is the smallest that \( \int_{0}^{4} f(x) \, dx \) can be? What is the largest it can be?

(c) Evaluate the integral \( \int_{-1}^{2} x^3 \, dx \) and interpret it as a difference of areas. Illustrate with a sketch.
4. Evaluate the integrals:

\( \int_{-2}^{3} (x^2 - 3) \, dx = \)

\( \int_{1}^{4} \left( \frac{4 + 6u}{\sqrt{u}} \right) \, du = \)

\( \int_{1}^{2} \left( \frac{x}{2} - \frac{2}{x} \right) \, dx = \)

\( \int_{1}^{e} \frac{x^2 + x + 1}{x} \, dx = \)

\( \int \frac{\sin(x)}{1 - \sin^2(x)} \, dx = \)

\( \int \tan(7) \, dx = \)