

score	possible	problem
	20	1
	20	2
	30	3
	30	4
	100	

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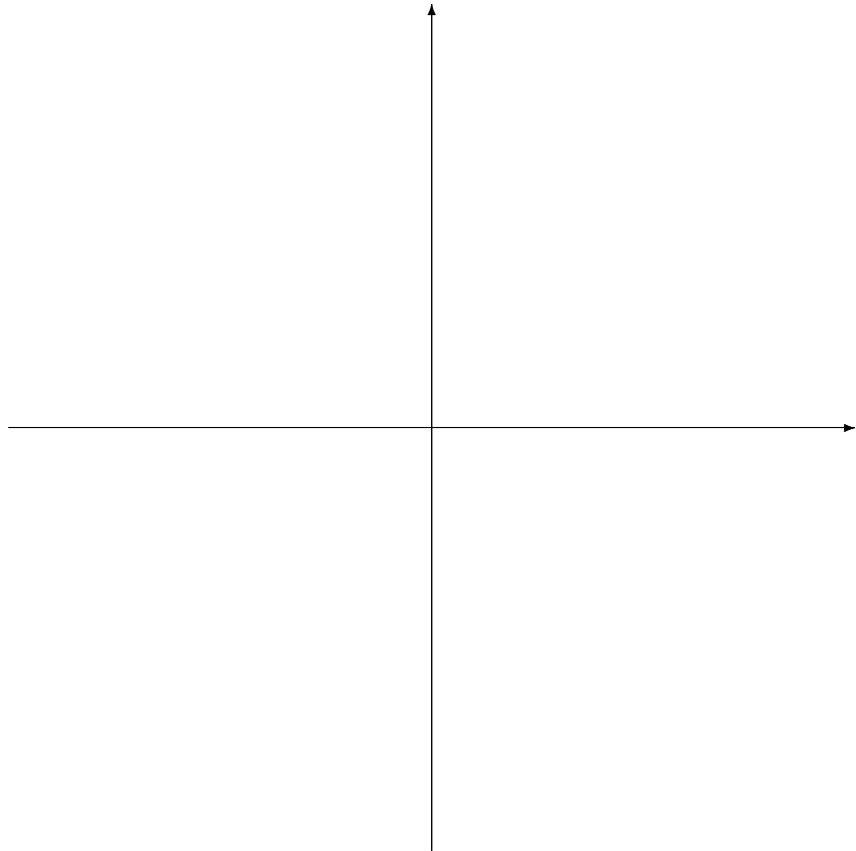
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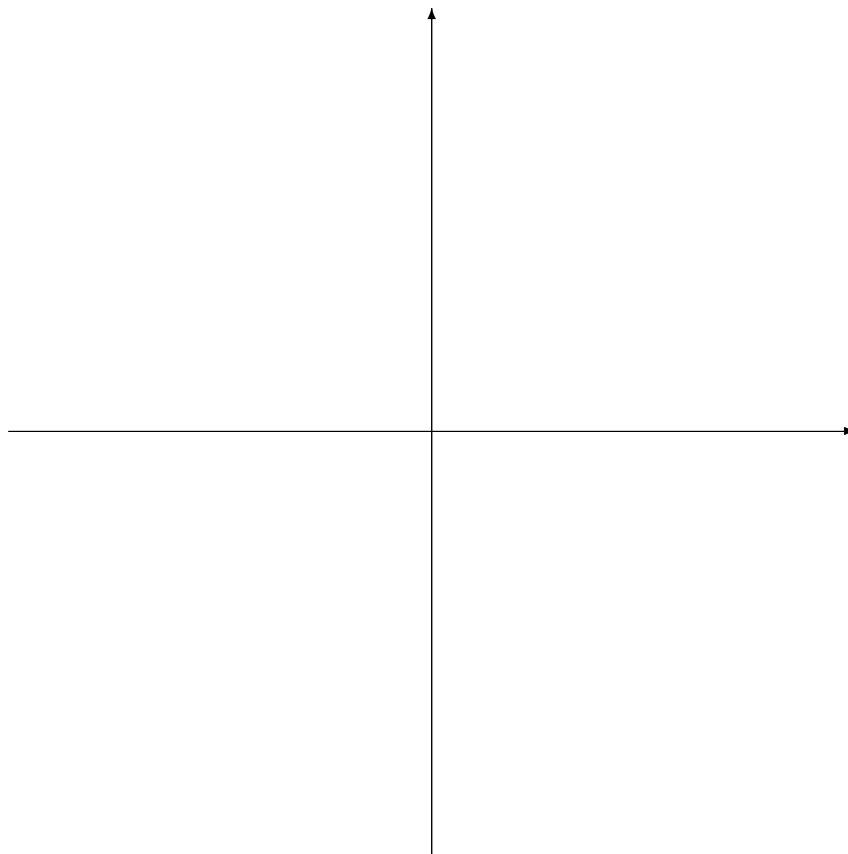
Work in groups of 3 or 4. Show your work. Acknowledge any help on these specific problems.

/20 1. Sketch the graph of a single function that has all of the following properties:

- (a)  $f$  is odd.
- (b)  $f'(x) < 0$  for  $0 < x < 2$ .
- (c)  $f'(x) > 0$  for  $x > 2$ .
- (d)  $f''(x) > 0$  for  $0 < x < 3$ .
- (e)  $f''(x) < 0$  for  $x > 3$ .
- (f)  $\lim_{x \rightarrow \infty} f(x) = -2$ .



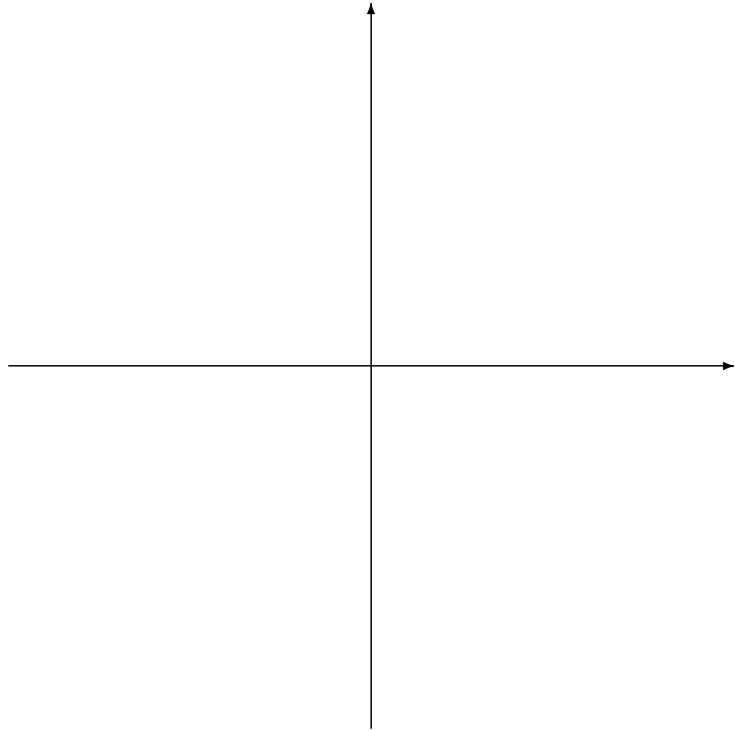
- /20      2. Sketch the graph of a single function that has all of the following properties:
- (a) Continuous and differentiable everywhere except at  $x = -3$ , where it has a vertical asymptote.
  - (b) A horizontal asymptote at  $y = 1$ .
  - (c) An  $x$ -intercept at  $x = -2$ .
  - (d) A  $y$ -intercept at  $y = 4$ .
  - (e)  $f'(x) > 0$  on the intervals  $(-\infty, -3)$  and  $(-3, 2)$ .
  - (f)  $f'(x) < 0$  on the interval  $(2, \infty)$ .
  - (g)  $f''(x) > 0$  on the intervals  $(-\infty, -3)$  and  $(4, \infty)$ .
  - (h)  $f''(x) < 0$  on the interval  $(-3, 4)$ .
  - (i)  $f'(2) = 0$ .
  - (j) An inflection point at  $(4, 3)$ .



/30 3. For the function

$$f(x) = 2 + 3x^2 - x^3$$

- (a) Find the  $y$ -intercept.
- (b) Find any asymptotes.
- (c) Find the intervals on which  $f$  is increasing or decreasing.
- (d) Find the local maximum and minimum values of  $f$ .
- (e) Find the intervals of concavity and the inflection points.
- (f) Use the information above to sketch the graph.



/30 4. For the function

$$f(x) = xe^{-x}$$

- (a) Find the  $x$ - and  $y$ -intercepts.
- (b) Find any asymptotes.
- (c) Find the intervals on which  $f$  is increasing or decreasing.
- (d) Find the local maximum and minimum values of  $f$ .
- (e) Find the intervals of concavity and the inflection points.
- (f) Use the information above to sketch the graph.

