/10

score	possible	problem
	20	1
	30	2
	30	3
	10	4
	10	5
	100	

Work in groups of 3 or 4. Show your work. Aknowledge any help on these specific problems.

/10 1. (a) Prove that  $\frac{d}{dx}\left(\cot^{-1}(x)\right) = -\frac{1}{1+x^2}$ .

(b) State L'Hôpital's Rule. Identify what are its assumptions (hypotheses) and what are its conclusions.

2. Compute the following limits:

/10 (a) 
$$\lim_{t \to 0} \frac{e^{2t} - 1}{\sin(t)} =$$

/10 (b) 
$$\lim_{x \to \infty} x^3 e^{-x^2} =$$

/10 (c) 
$$\lim_{x \to 0^+} x^{\sqrt{x}} =$$

3. Compute the following derivatives:

(a) 
$$\frac{d}{dx}\sqrt{\arctan(3x)} =$$

(b) 
$$\frac{d}{dx}\cos(\cosh(\cos^{-1}(x))) =$$

(c) 
$$\frac{d}{dx} \frac{\cot^{-1}(1+x^2)}{\operatorname{sech}(5x)} =$$

/10 4. If an electrostatic field E acts on a liquid polar dielectric, the net dipole moment P per unit volume is

$$P(E) = \frac{\cosh(E)}{\sinh(E)} - \frac{1}{E}.$$

Show that  $\lim_{E\to 0^+} P(E) = 0$ .

5. Show that  $\cosh^2(x) - \sinh^2(x) = 1$ .

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