Société de Calcul Mathématique SA
Outils d'aide à la décision

Fédération Française des Jeux Mathématiques

Mathematical Competitive Game 2013-2014

Checking an Industrial Process

Fédération Française des Jeux Mathématiques
(French Federation of Mathematical Games)

and

Société de Calcul Mathématique SA

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I. Presentation of the Games

The "Mathematical Games", jointly organized by FFJM and SCM, have existed for five years; the previous ones were:

- In 2008-2009, conception of a bus transportation network in a city, in partnership with Veolia Transport;

- In 2009-2010, conception of an electricity distribution network, in partnership with RTE (Réseau de Transport d'Electricité);

- In 2011-2012, search for the best itinerary by a car, in partnership with the newspaper Auto Plus;

- In 2012-2013, fighting forest fires in Siberia, in partnership with the Paris Firemen Brigade.

They deal with the resolution of a "real life" problem, that is a problem of general concern, but simplified in its mathematical contents. Still, the resolution typically requires several months of work.

Candidates may compete individually or as groups, for instance high school classes, or college students, or university students, preparing a "memoir" for the end of their studies.

Two categories of prizes are given:

Individual prizes:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

Prizes for groups:
For the winner: 500 Euros
For the second: 200 Euros
For the next three: 100 Euros each.

The total amount of prizes is therefore 2 000 Euros. The best solutions are published on the web site of FFJM, on the web site of SCM, and on the web sites of our partners. The official announcement of the results and the ceremony for prizes occur during the "Salon de la Culture et des Jeux Mathématiques" (Fair for Mathematical Culture and Games), which is held in Paris, each year, during the month of May.

The winners, previous years, gained considerable notoriety, both in the press and television in their respective countries.

II. The 2013-2014 Prize

A. General presentation of the subject

It deals with the general concern of "checking" the quality of an industrial process, here the production of metal cylinders. There are some requirements, defined by the Safety Authorities; these requirements are of probabilistic nature. Of course, one cannot check all pieces, because checking is costly and is usually of destructive nature: the pieces used for checking are destroyed. So a sample is chosen, and, for each designated part, some measurements are made, also at random places in the part.

Quite often, the Industrial Company has its own requirements, which are of higher nature: the Company wants the pieces to be of good quality, but also of constant quality, which is not the same thing. Therefore, the Company defines its own testing procedures, which are supposed to be stricter than the rules of the Safety Authorities: the Company cannot afford to have its products rejected.

B. Technical description of the process

An Industrial Company produces disks, made of stainless steel. These disks are used in severe environments, for instance in Power Plants, and the Safety Authorities want to make sure that these disks are of constant, acceptable, quality.

What the Company produces is the so-called "austenitic stainless steel" (in short ASS), which should contain 18% chromium and 8% nickel, the percentages being computed as fractions of the total mass. This is the classical "304 ASS", a standard alloy.

The disks have 0.5 m of radius and 0.5 m of height. Each disk should be made of an homogeneous alloy, which means that, hopefully, the proportions of steel, chromium and nickel should be the same everywhere in the disk, and should be the same in all disks.

The requirements of the Safety Authorities are :

All the time:
- Chromium: between 16.95 % and 19.10 %
  and
- Nickel: between 6.94 % and 9.10 %

95% of the time:
- Chromium : between 17 % and 19 %
  and
- Nickel: between 7 % and 9 %.

The Company wants to comply with these requirements, of course, but wants to go further. They want to define stricter requirements. This is quite common in the Industry, for two reasons:

– The Industry knows that the regulations are going to change, and wants to anticipate these changes;

– The Industry wants to deliver goods which are of good quality, of course, but more specifically which are of constant quality.

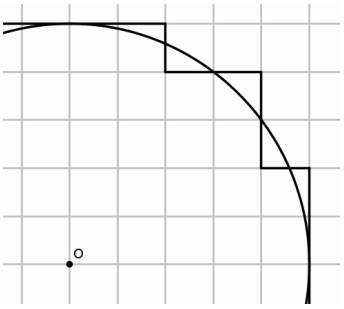
Therefore, the Industry defines its own requirements, as follows :

99 % of the time:
– Chromium: between 16.96 % and 19 %
and
– Nickel: between 6.95 % and 9.05 %

95% of the time:
– Chromium: between 17.1 % and 18.9 %
and
– Nickel: between 7.1 % and 8.9 %

C. Discretization of the cylinders

For checking purposes, each cylinder is divided into small cells, the following way:



Each cell is 0.1 m x 0.1 m x 0.1 m. There are five layers of 0.1 m each, for a total height of the cylinder of 0.5 m. On each layer, the disk is replaced by 80 cells according to the picture above (showing only a quarter of the disk). If the unit is 0.1 m, the cells which are kept are those which satisfy:

$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2 \le 25$$

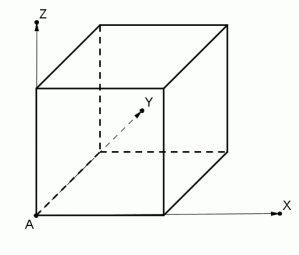
that is, the center of which is inside the disk.

This representation of the cylinder by 400 cubic cells is accepted both by the Company and by the Safety Authorities.

Each cell is considered as a unit for testing and is assumed to be homogeneous.

D. Mathematical representation:

The layers are numbered from 0 (bottom, altitude 0) to 4 (top). The unit is 0.1 m. The cells are numbered by the coordinates of their West-South-Bottom corner.



If (c_1, c_2, c_3) are the coordinates of the center of a cell, the coordinates of the point A which characterizes the cells are $\left(c_1 - \frac{1}{2}, c_2 - \frac{1}{2}, c_3 - \frac{1}{2}\right)$.

E. Production rules and checking rules

1. Production

The Company produces 10 000 cylinders: 1 000 each day, during ten days.

2. Safety Authorities checking

The Safety Authorities will pick up 100 cylinders at random (uniform law) and in each cylinder 10 cells at random (uniform law). The requirements defined above refer to this sample of 1 000 cells. For instance, saying "95% of the time" means, for the Safety Authorities, that 950 cells at least must satisfy the corresponding requirement.

3. Company checking

The Company performs two types of checking:

First, Nondestructive checking: each day, 10 cylinders are taken at random, put aside, and their upper and lower surfaces (disks) are checked electronically. This concerns only 2 x 80 cells (not the side of the cylinder, only the top and bottom); 10 cells are chosen at random among the top 80 cells and 10 are chosen at random among the bottom 80 cells.

These 20 cells are checked by a process which does not damage the cylinder. The Company checks a total of 2 000 cells. These cylinders, after checking, are put back in the production.

Second, Destructive checking: each day, 10 cylinders are taken at random, put aside, and decomposed into cells; in each cylinder, 20 cells are chosen at random and checked. Since the production lasts for 10 days, the Company checks 100 cylinders and 2 000 cells.

Checking by the Company comes first, and checking by the Safety Authorities comes after. The Safety Authorities do not know the result of the checking made by the Company. After the checking by the Company, the non-damaged cylinders are put back in the production, and the damaged ones are replaced, so the Safety Authorities truly have at their disposal a sample of 10 000 pieces.

F. Penalties and fines

Each time a result is found by the Safety Authorities, which does not comply with the established rules, the Company must pay a fine, as follows:

- If a cell is found, which is not between 16.95 % and 19.10 % of chromium, or which is not between 6.94 % and 9.10 % of nickel, the Company must pay a fine of 1 million Euros, for each such cell.

- If the total number of cells, among the 1 000 cells which are checked, satisfying both between 17 % and 19 % of chromium and between 7 % and 9 % of nickel, is strictly smaller than 950, then the Company pays also a fine of 1 million Euros.

G. Results of the checking made by the Company

These results are given in the Excel file (Excel 2003 format):
data_mathematical_game_2013_2014.xls
sheets(2): non-destructive testing
sheets(3): destructive testing

see http://scmsa.eu/archives/data_mathematical_game_2013_2014.xls

The Company observes that, from its own point of view, the results of the testing is satisfactory.

H. Question

What is the expected fine (expectation of the probability law) to be paid by the Company ?

I. Remark

In an industrial process, different cells of the same object cannot be considered as independent, nor different objects produced the same day. Differences in composition may come from small initial differences in the proportions of each metal, and from small differences in the way the process is performed (for instance, uneven heating and cooling, and so on).

III. Participation rules

The game starts on November 1st, 2013 and ends on April 30th, 2014. Prizes will be given in May 2014, during the "Salon des Jeux Mathématiques", in Paris.

Participants should send their solution, in pdf format, in English or in French, no later than April 30th, 2014, to the email address: **ffjm@wanadoo.fr.**

No preliminary registration is required. Everyone can participate.