

**Math 2301****Guide for Test 2**

Here are some sample questions from sections 2.5–2.8, 3.2, 3.3, 3.5, and 3.6. The actual test will be significantly shorter.

1. Compute the following derivatives:

(a)  $\frac{d}{dx} [(x^9 + 2x^{1/3} + x^5 + 3)^4] =$

(b)  $y = (3 + x^4)^8 x^3 \Rightarrow \frac{dy}{dx} =$

(c)  $\frac{d}{dx} [5 \tan(x^2 \sin(x^3 + 7x))] =$

(d)  $D_x \left[ ((x^9 + x^8 + x^5 + 3)(1 + 2x^2 + x^3 - 4x^4) + 1)^9 \right] =$

[Tests chain rule (2.5) and earlier differentiation formulas.]

2. Use implicit differentiation to find an equation for the tangent line to the curve defined by

$$x^2 + 2xy - y^2 + x = 2$$

at the point  $(1, 2)$ .

[Tests implicit differentiation (2.6).]

3. If  $y = 4x^3$  and  $\frac{dx}{dt} = 5$ , find  $\frac{dy}{dt}$  when  $x = 2$ .

[Tests related rates (2.7).]

4. A kite 100 feet above the ground moves horizontally at a speed of 8 feet per second. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string has been let out?

[Tests related rates (2.7).]

5. Use a linear approximation (or differentials) to estimate  $\sqrt{99.8}$ .

[Tests linear approximation (2.8).]

6. One side of a right triangle,  $x$ , is measured at 10 inches with a possible error of 0.5 inches. The angle opposite  $x$  is  $30^\circ$ .

(a) Use differentials to estimate the maximum error in calculating the hypotenuse based on the measurement of  $x$ .

(b) What is the percentage error?

[Tests the application of differentials in 2.8.]

7. Find the inverse function of  $f(x) = \frac{x+1}{2x+1}$ .

[Tests concept of inverse function (3.2).]

8. For the function

$$f(x) = \sqrt{x^3 + x^2 + x + 1}$$

find  $(f^{-1})'(2)$ .

[Tests theorem for the derivative of the inverse function (3.2).]

9. (a) Solve the equation  $e^{2x} - 6e^x = -5$ .

(b) Solve the equation  $\ln(x) + \ln(x - 1) = 1$  for  $x$ .

(c) Find  $\lim_{x \rightarrow 0^+} \ln(\sin(x))$ .

[Tests understanding of  $e$  and  $\ln$  (3.2).]

10. Prove that  $\cosh(x + y) = \cosh(x) \cosh(y) + \sinh(x) \sinh(y)$ .

[Tests hyperbolic functions (3.6).]

11. Compute the following derivatives:

(a)  $f(x) = \sinh(\cosh(x)) \Rightarrow f'(x) =$

(b)  $f(x) = \arctan(x) \Rightarrow f'(x) =$

(c)  $f(x) = \ln(x) \Rightarrow f'(x) =$

(d)  $f(x) = \log_3(x) \Rightarrow f'(x) =$

(e)  $f(x) = \frac{\arctan(\log_3(7x))}{x^4 + 2x} \Rightarrow f'(x) =$

(f)  $f(x) = e^x \Rightarrow f'(x) =$

(g)  $f(x) = 3^x \Rightarrow f'(x) =$

(h)  $f(x) = x^3 \Rightarrow f'(x) =$

[Tests differentiation formulas for exponentials (3.3), logarithms (3.3), inverse trigonometric functions (3.5), and hyperbolic functions (3.6).]

12. Find the derivative of  $y = \frac{x^x \sin(2x)(x^5 - 7x)^6}{(\sqrt{x^9 + 1})3^x}$ .

[Tests logarithmic differentiation (3.3).]

13. Find an equation of the tangent line to the graph of  $y = 3 \arccos(x/2)$  at  $(1, \pi)$ .

[Tests inverse trigonometric functions and their derivatives. From spring 2013 final exam.]