Math 2301

Guide for Test 1

Here are some sample questions from sections 1.3–1.6 and 2.1–2.4. The actual test will be significantly shorter.

1. Consider the function
   \[ f(x) = \begin{cases} 
   x^2 & \text{if } x \leq -2 \\
   Ax & \text{if } x > -2 
   \end{cases}, \]
   where \( A \) is some constant.
   (a) Find \( \lim_{x \to -2^-} f(x) \). Is \( f \) continuous from the left at \( x = -2 \)?
   (b) What value of \( A \) would make \( f \) continuous at \( x = -2 \)?
   (c) Using the value of \( A \) that you just found, graph \( f \).

   [Tests one-sided limits (Sections 1.3 and 1.4) and the concept of continuity (1.5).]

2. Use the Intermediate Value Theorem to show that the equation \( x^2 = \cos(x) \) has a solution.
   [Tests the concept of continuity (1.5) and its use in the Intermediate Value Theorem.]

3. Compute the following limits. If you use the squeeze theorem, then indicate the two functions that you are using to squeeze.
   (a) \( \lim_{x \to 2} \frac{x - 2}{x^2 - 5x + 6} \)
   (b) \( \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \)
   (c) \( \lim_{x \to 0} x^2 \cos(3/x) \)
   (d) \( \lim_{h \to 0} \frac{x^2 - (x - 2h)^2}{h} \)
   (e) \( \lim_{t \to 0} \frac{\frac{1}{5+t} - \frac{1}{5}}{t} \)
   (f) \( \lim_{x \to +2} \frac{x + 2}{x^2 - 5x + 6} \)
   (g) \( \lim_{x \to -\infty} \frac{3x^3 - 4}{2x^3 - 2} \)
   (h) \( \lim_{x \to \infty} \cos(1/x) \)
   (i) \( \lim_{x \to \infty} (x - x^2) \)

   [Tests the computation of limits (1.4, 1.6).]

4. Let \( f(x) = -x^2 + 3 \).
   (a) State the definition of the derivative as a limit.
   (b) Using this definition, compute \( f'(x) \).
   (c) Find the equation for the tangent line at \( x = 2 \).
(d) Graph \( f(x) \) and the tangent line.

[Test computation of the derivative from the limit definition (2.2), the use of the derivative to get the slope of the tangent line (2.1), and the construction of a tangent line (2.1).]

5. Find the limit. What derivative does this limit represent?

\[
\lim_{h \to 0} \frac{(x + h)^3 - x^3}{h}
\]

[Test computation of limits (1.4) and definition of the derivative (2.2). Taken from spring 2013 final exam.]

6. Sketch the graph of a function \( f \) for which:
   It has a removable discontinuity at \( x = 1 \), \( \lim_{x \to 2^-} f(x) = +\infty \), \( \lim_{x \to 2^+} f(x) = -1 \), \( \lim_{x \to +\infty} f(x) = 3 \), \( f'(x) > 0 \) everywhere it exists.

[Test concept of limits (1.3, 1.6), continuity (1.5), and derivative (2.1, 2.2). Taken from spring 2013 final exam.]

7. Find values for \( m \) and \( b \) so that \( f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\ mx + b & \text{if } x > -2 \end{cases} \) is differentiable at \( x = -2 \).

[Test the concept of differentiability (2.2).]

8. Sketch the graph of a function \( g \) for which \( g(0) = g'(0) = 0 \), \( g'(-1) = -1 \), \( g'(1) = 3 \), and \( g'(2) = 1 \).

[Test graphical understanding of the derivative (2.2).]

9. The graph of a function \( f \) is given in each part below. On the same axes, sketch the graph of \( f' \).

[Test graphical understanding of the derivative (2.2).]

10. Compute the following derivatives:

   (a) \( f(x) = 2 + x + \frac{3}{x} - \sqrt{x} - 5x^7 + x^{3/4} \Rightarrow f'(x) = \)

   (b) \( \frac{d}{dx} [3 \sin(x) + \cot(x) - \sin(7)] = \)

   (c) \( \frac{D}{x} [(x^9 + x^8 + x^5 + 3)(1 + 2x^2 + 9x^3 - 4x^4)] = \)

   (d) \( y = \frac{x^3 + x}{x} \Rightarrow \frac{dy}{dx} = \)

[Test the basic differentiation formulas (2.3) and product and quotient rules (2.4).]