The final exam is on Wednesday May 1 12:20–2:20 pm in our regular classroom. No books or notes are allowed. Here are some sample questions.

1. Question directly from Part I test.
2. Question directly from Part II test.
3. Question directly from Part III test.
4. Gaussian Elimination:
   (a) State and prove the three strokes of luck that make Gaussian elimination work so nicely.
   (b) Your classmates have nearly come to blows debating whether one should solve a linear system $Ax = b$ using QR or using LU (with pivoting). Choose a side and argue your case.
5. Cholesky Factorization:
   Give a constructive proof of:
   Every hermitian positive definite matrix $A \in \mathbb{C}^{m\times m}$ has a unique Cholesky factorization $A = R^*R$ where R is upper triangular and has $r_{jj} > 0$ for all j.
6. Eigenvalues:
   (a) Prove that every square matrix A has a Schur factorization.
   (b) Show how to reduce any square matrix to upper Hessenberg form using unitary similarity transformations. Determine the operation count (to leading order).
   (c) Why must every eigenvalue solver be iterative?
   (d) Show that the Rayleigh quotient gives a quadratically accurate estimate of an eigenvalue.
   (e) Give pseudo-code for the ("pure") QR algorithm. Explain what it is intended to produce and explain why it does so.
   (f) The QR algorithm with Rayleigh quotient shift chooses shift $\mu^{(k)} = A^{(k)}_{mm}$. What happens if you instead choose shift $\mu^{(k)} = A^{(k)}_{jj}$ for some $1 \le j < m$?
7. Some really cool open-ended problem (if I can think of one).