The test is Wednesday March 13. Here are some sample questions, some of which are from old tests. No books or notes are allowed.

1. Projectors:
   (a) Prove that if P is a projector then $\text{range}(I - P) = \text{null}(P)$.
   (b) Prove that a projector P is an orthogonal projector if and only if $P = P^*$.
2. QR:
   (a) State the formal definition of the full QR factorization for a matrix $A \in \mathbb{C}^{m\times n}$ with $m \geq n$.
   (b) Prove that every $A \in \mathbb{C}^{m\times n}$ with $m \geq n$ has a full QR factorization.
3. Modified Gramm-Schmidt:
   (a) Describe the modified Gramm-Schmidt algorithm and show that it produces a QR factorization.
   (b) Give the algorithm as pseudocode and compute the number of flops needed.
4. Householder:
   (a) Define a Householder reflector and show that it does what you say it does.
   (b) Describe the Householder algorithm and show that it produces a QR factorization.
   (c) Give the algorithm as pseudocode and compute the number of flops needed.
5. Least squares:
   (a) Fill in the blank and Prove:
   Let $A \in \mathbb{C}^{m\times n}$ with $m \geq n$ and $b \in \mathbb{C}^m$ be given. A vector $x \in \mathbb{C}^n$ minimizes the residual norm $\|r\|_2 = \|b - Ax\|_2$ if and only if
   _____.
   The solution x is unique if and only if A has full rank.
   (b) Show how to solve a least squares problem using the QR factorization.
   (c) Show how to solve a least squares problem using the SVD.