Here are some sample questions.

1. For each formula, determine which values of x may lead to loss of significance and suggest a better way to compute the same quantity for those x.

   (a) $\sqrt{x^2 + 1} - x$

   (b) $\dfrac{1 - x}{1 + x} - \dfrac{1}{3x + 1}$

2. Define *condition number*. If the conditon number is 10^5, what does that mean?

3. (a) State Taylor's theorem on the approximation of a function about a point using a polynomial of degree k and one form of the remainder.

   (b) Prove this theorem.

4. We would like to find the root of $f(x) = ***$ in the vicinity of $x = **$.

   (a) Set up and perform two iterations of the Bisection method.

   (b) Set up and perform two iterations of Newton's method.

   (c) Set up and perform two iterations of the Secant method.

   (d) Describe the advantages and disadvantages of the three methods.

5. (a) State the conditions under which Newton's method for solving $f(x) = 0$ will have quadratic convergence.

   (b) Prove that Newton's method does indeed have quadratic convergence under these conditions.

6. Consider the fixed-point iteration

$$x_{n+1} = **** \tag{1}$$

   (a) Apply the iteration twice, starting at $x_0 = **$.

   (b) Determine the fixed point(s) of this iteration.

   (c) Draw a cobweb plot to show which initial x_0 will converge to which fixed point, and which will diverge.

   (d) For one of the fixed points, determine the order of convergence.

7. For each of the following sequences, what is the observed rate of convergence?

   (a) $1, 1/2, 1/4, 1/8, \ldots$

   (b) $1, 1/2, 1/4, 1/16, \ldots$

   (c) $10^{-3}, 10^{-5}, 10^{-9}, 10^{-17}, \ldots$