Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Write the IVP: $\theta'' + .5\theta' + \sin\theta = \sin 2t$, $\theta(0) = 1$, $\theta'(0) = 0$ as a system of first order equations. Give all the MATLAB commands needed to solve this IVP on the interval $0 \le t \le 10$.

2. Describe RK45. What is the command for it in MATLAB?

3. What is variable step size? How is it implemented RK45?

4. Write a MATLAB **function** program to do n steps of the Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, on the time interval $[a, b]$ with $\mathbf{x}(a) = \mathbf{x}_0$. Include comments. Let the first line be:
`function [T, X] = myeuler(f,x0,a,b,n).`

5. Write a MATLAB **function** program to do n steps of the modified Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, on the time interval $[a, b]$ with $\mathbf{x}(a) = \mathbf{x}_0$. Let the first line be:
`function [T, X] = mymodeuler(f,x0,a,b,n).`

6. (a) Derive the explicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$ on the interval $x \in [0, L]$ with boundary conditions $u(0, t) = a$, $u(L, t) = b$, and $u(x, 0) = f(x)$.
   (b) When and why does the explicit finite difference method for the heat/diffusion equation become unstable?
   (c) Explain how to incorporate an insulated boundary at $x = L$.

7. Derive the implicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$.

8. (a) Set up the finite difference equations for the BVP: $u_{xx} + u_{yy} = f(x, y)$, on the rectangle $0 \le x \le a$ and $0 \le y \le b$, with $u = 0$ on all the boundaries. Explain how the difference equations could be solved as a linear system.
   (b) Explain how to incorporate an insulated boundary at $x = a$.

9. Set up the finite difference equations for the BVP: $u_{rr} + \frac{1}{r}u_r = f(r)$, on the interval $0 \le r \le R$, with $u(R) = 4$ and $u_r(0) = 0$. Explain how to avoid the problem at $r = 0$.

10. Explain how to incorporate an insulated boundary in a finite difference method.

11. What are main differences between the Finite Difference Method and Finite Elements Method?

12. If $U(x) = \sum_{j=1}^{n} C_j \Phi_j(\bar{x})$ is a finite element solution, what is the meaning of C_j? Describe how the C_j are obtained.

13. Explain why order matters in engineering problems.