Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. (a) Using pivoting if appropriate, find the LU decomposition of
$$A = \begin{bmatrix} 2 & 1 \\ 6 & 2 \end{bmatrix}.$$

   (b) Using your LU decomposition, solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = [5, 3]'$.

2. Write a MATLAB **function** program that solves the linear systems $A\mathbf{x}_1 = \mathbf{b}_1$ and $A\mathbf{x}_2 = \mathbf{b}_2$ using LU decomposition. Let A, b1, and b2 be the inputs and x1 and x2 be the outputs. Include comments.

3. Write a MATLAB **function** program to that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A, $\mathbf{b}$ and *tol* be the inputs and $\mathbf{x}$ the output. If the error (residual) is not less than *tol*, then display a warning.

4. (a) Find the eigenvalues and eigenvectors of the matrix
$$A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}.$$

   (b) Perform one iteration of the power method, starting with $[1, 1/2]'$. What approximate eigenvalue and eigenvector did you get? If you kept iterating, what would you get eventually?

   (c) Describe how you would use the inverse power method to find one eigenvalue and eigenvector. Which one would it find?

5. Suppose $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$. Using $\mathbf{v}_0 = (1, 1)'$ as the starting vector do 2 iterations of the Power Method for A. What do the results mean?

6. Write a MATLAB **function** program to do n iterations of the Power Method. Let the matrix A and n be inputs and let [e v] (the eigenvalue and eigenvector) be the outputs.

7. Write a MATLAB **script** program that will use Newton's method to find a root of the system of functions $f_1(x, y) = x^3 - 7y^2 + 1$ and $f_2(x, y) = 5y^3 + x - 1$ starting from the initial guess $(0, 0)$. Include comments.

8. Write a MATLAB **script** program that will use Newton's method to find a solution to the system of equations
$$x^3 - 7y^2 + 1 = 0$$
$$5y^3 + x + 2z - 2 = 0$$
$$5z^3 + x - y^2 - 3 = 0$$
starting from the initial guess $(x, y, z) = (1, 1, 1)$. Include comments.

9. What is the condition number of a matrix? How do you find it with MATLAB? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?

10. Give the MATLAB commands, or sequences of commands for solving a linear system $A\mathbf{x} = \mathbf{b}$ in as many ways as you know. Which of these are the worst and best?

11. What is the command in MATLAB to produce the eigenvalues and eigenvectors of a matrix. Which method does it use? What will be the form of the output?

12. Write a MATLAB **function** program that takes an input n, produces a random $n \times n$ matrix A and random vector $\bar{b}$, solves $A\bar{x} = \bar{b}$ (using the built in command) and outputs the residual (number).