Math 266B **Guide for Test 1**

Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Compute the following limits. If you use the sandwich theorem or L'Hôpital's rule, then say so.

   (a) $\lim_{x\to 0} \frac{e^x - 1}{3x} =$

   (b) $\lim_{x\to 0^+} \frac{e^x}{3x} =$

   (c) $\lim_{x\to\infty} \frac{(\ln(x))^2}{x^2} =$

   (d) $\lim_{x\to\infty} x^{1/x} =$

2. Find the function f for $x > 0$ that has $f''(x) = x^{-2}$, $f(1) = 0$, and $f(2) = 0$.

3. Based on the definition of the definite integral, approximate

$$\int_1^3 \sin((x+2)^3)\, dx$$

   using $n = 4$ rectangles.

4. If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$, what is $\int_1^4 f(x)dx$?

5. If $f(1) = 3$ and $f'(x) \geq 0$ for all x, what is the smallest that $\int_1^5 f(x)dx$ can be? What is the largest it can be?

6. Compute

   (a) $\int \left(e^x + x^{-3} + x^{1/3} + 3x^{-1}\right) dx$

   (b) $\int_1^3 \sin(3)dx =$

   (c) $\int_0^\pi \sin(x)dx =$

   (d) $\int_5^5 \arctan(x)dx =$

   (e) $\int \frac{x^2 - 3}{x}\, dx =$

   (f) $\int \left(3\csc^2(x) - \frac{5}{1+x^2}\right) dx$

   (g) $\frac{d}{dx}\int_9^x \frac{\tan(t)}{\sqrt{t}}dt =$

   (h) $\frac{d}{dx}\int_4^2 \frac{\sqrt{t+1}}{\ln t}dt =$

7. (a) Compute the area of the region enclosed by the curves $y = (x-1)^2$ and $y = x + 1$.

   (b) Compute the volume obtained by rotating this region about the x-axis.