Math 444/544

Guide for Test 2

Test 2 is on Monday 1 November; note the change in day. I attempted to outline my test and give you an outline, but there is no warranty.

1. Suppose we wish to make an approximation \( \int_x^* f(x)dx \approx c_1 f(*) + c_2 f(x_2) \).
   (a) Determine the best values of \( c_1, c_2, \) and \( x_2 \).
   (b) Apply your method to \( f(x) = *** \) and compute the error.

2. Suppose that we have a method \( N(h) \) to approximate some quantity \( M \), and we know \( M = N(h) + k_1 h^2 + k_2 h^3 + \cdots \). Given the values \( N(*) = ** \) and \( N(**) = *** \), find the best approximation for \( M \).

3. To numerically solve an initial value ODE \( y'(t) = f(t, y(t)) \) with \( y(t_0) = y_0 \), the following method is proposed:
   \( y_{n+1} = ** \text{RK type}**. \)
   (a) Determine (and prove) the order of this method.
   (b) Determine the stability domain of this method.
   (c) Given \( f(t, y) = *** \), \( y(t_0) = y_0 = ** \), and \( h = *** \), compute \( y_2 \).

4. To numerically solve an initial value ODE \( y'(t) = f(t, y(t)) \) with \( y(t_0) = y_0 \), the following method is proposed:
   \( y_{n+1} = ** \text{multistep type}**. \)
   (a) Determine (and prove) the order of this method.
   (b) Determine if the method is stable.
   (c) Given \( f(t, y) = *** \), \( y(t_0) = y_0 = ** \), \( y(t_0 + h) = y_1 = *** \), and \( h = *** \), compute \( y_3 \).

5. Math 444 students: Make sure you wrote your name on the test.

Math 544 students: Prove the following theorem:

Theorem: Suppose \( x_1, x_2, \ldots, x_n \) are the roots of the \( n \)th Legendre Polynomial \( P_n(x) \) and that for each \( i = 1, 2, \ldots, n \), the numbers \( c_i \) are defined by
\[
c_i = \int_{-1}^1 \prod_{j=1, j \neq i}^n \frac{x - x_j}{x_i - x_j} dx.
\]

If \( P(x) \) is any polynomial of degree less than \( 2n \), then
\[
\int_{-1}^1 P(x)dx = \sum_{i=1}^n c_i P(x_i).
\]