• The test is in class on Friday 7 May and covers Chapters 3, 5, and 6. You must notify me in advance if you have to miss the test.

• Only Math 510 students are responsible for the material in Sections 3.6 and 5.7. They will have an extra question to prove a lemma or theorem in those sections or a theorem from another section.

Here are some sample test questions/topics. Things written in [brackets] are comments.

1. [Probably a 2 × 2 with some of these questions and a 3 × 3 with others of these questions.] Let \( A = \begin{bmatrix} 1 & 2 \\ -4 & 3 \end{bmatrix} \) and \( b = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \).

   (a) Compute the determinant of \( A \) using [cofactor expansion or elementary row operations and pivotal condensation].

   (b) Compute \( A^{-1} \) using elementary row operations (Gaussian Elimination).

   (c) Compute \( A^{-1} \) using the adjugate.

   (d) Check that the matrices you computed above satisfy the definition of the inverse of \( A \).

   (e) Use the inverse you computed above to solve \( Ax = b \).

   (f) Compute the LU decomposition of \( A \) and use it to solve \( Ax = b \).

   (g) Use Cramer’s rule to solve \( Ax = b \).

   (h) Find all the eigenvalues of \( A \), with multiplicities.

   (i) Find the eigenvectors of \( A \). [There may be fewer linearly independent eigenvectors than the size (order) of \( A \).]

   (j) Show that the eigenvectors that you found above satisfy the definition of an eigenvector.

2. [More theoretical questions.]

   (a) [properties of the inverse from section 3.4] Show that \( (BC)^{-1} = C^{-1}B^{-1} \).

   (b) [properties of determinant from section 5.3] Show that the determinant of an upper triangular matrix is the product of the elements on the main diagonal.

   (c) [properties of eigenvalues and vectors from 6.4] Show that if \( \lambda \) is an eigenvalue of \( B \) and \( B \) is invertible, then \( 1/\lambda \) is an eigenvalue of \( B^{-1} \).