Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Write the IVP: $\theta'' + .5\theta' + \sin\theta = \sin 2t$, $\theta(0) = 1$, $\theta'(0) = 0$ as a system of first order equations. Give all the MATLAB commands needed to solve this IVP on the interval $0 \le t \le 10$.

2. (a) Derive the explicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$ on the interval $x \in [0, L]$ with boundary conditions $u(0, t) = a$, $u(L, t) = b$, and $u(x, 0) = f(x)$.
   (b) When and why does the explicit finite difference method for the heat/diffusion equation become unstable?

3. If $U(x) = \sum_{j=1}^{n} C_j \Phi_j(\bar{x})$ is a finite element solution, what is the meaning of C_j? Describe how the C_j are obtained.

4. Explain why order matters in engineering problems.

5. Write a MATLAB program to do n steps of the Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, on the time interval $[a, b]$ with $\mathbf{x}(a) = \mathbf{x}_0$. Include comments. Let the first line be:
   `function [T, X] = myeuler(f,x0,a,b,n)`.

6. Write a MATLAB program to do n steps of the modified Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, on the time interval $[a, b]$ with $\mathbf{x}(a) = \mathbf{x}_0$. Let the first line be:
   `function [T, X] = mymodeuler(f,x0,a,b,n)`.

7. Describe RK45. What is the command for it in MATLAB?

8. What is variable step size? How is it implemented RK45?

9. Derive the implicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$.

10. Set up the finite difference equations for the BVP: $u_{xx} + u_{yy} = f(x, y)$, on the rectangle $0 \le x \le a$ and $0 \le y \le b$, with $u = 0$ on all the boundaries. Explain how the difference equations could be solved as a linear system.

11. Set up the finite difference equations for the BVP: $u_{rr} + \frac{1}{r}u_r = f(r)$, on the interval $0 \le r \le R$, with $u(R) = 4$ and $u_r(0) = 0$. Explain how to avoid the problem at $r = 0$.

12. Explain how to incorporate an insulated boundary in a finite difference method.

13. What are main differences between the Finite Difference Method and Finite Elements Method?

14. How are the boundary and interior values of the finite element solution obtained?