Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. (a) Using pivoting if appropriate, find the LU decomposition of

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}.$$

   (b) Using your LU decomposition, solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = [3, 5]'$.

2. Write a MATLAB function program that solves the linear systems $A\mathbf{x}_1 = \mathbf{b}_1$ and $A\mathbf{x}_2 = \mathbf{b}_2$ using LU decomposition. Let A, $\mathbf{b}_1$, and $\mathbf{b}_2$ be the inputs and $\mathbf{x}_1$ and $\mathbf{x}_2$ be the outputs. Include comments.

3. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}.$$

   (b) Describe how you would use the inverse power method to find one eigenvalue and eigenvector. Which one would it find?

4. Write a MATLAB script program that will use Newton's method to find a root of the system of functions $f_1(x, y) = x^3 - 7y^2 + 1$ and $f_2(x, y) = 5y^3 + x - 1$ starting from the initial guess $(0, 0)$. Include comments.

5. What is the condition number of a matrix? How do you find it with MATLAB? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?

6. Write a MATLAB program to that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A, $\mathbf{b}$ and *tol* be the inputs and $\mathbf{x}$ the output. If the error (residual) is not less than *tol*, then display a warning.

7. Suppose $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$. Using $\mathbf{v}_0 = (1, 1)'$ as the starting vector do 2 iterations of the Power Method for A. What do the results mean?

8. Write a MATLAB program to do n iterations of the Power Method. Let the matrix A and n be inputs and let $[e\ v]$ (the eigenvalue and eigenvector) be the outputs.

9. Give the MATLAB commands, or sequences of commands for solving a linear system $A\mathbf{x} = \mathbf{b}$ in as many ways as you know. Which of these are the worst and best?

10. What is the command in MATLAB to produce the eigenvalues and eigenvectors of a matrix. Which method does it use? What will be the form of the output?

11. Write a MATLAB function program that takes an input n, produces a random $n \times n$ matrix A and random vector $\bar{b}$, solves $A\bar{x} = \bar{b}$ (using the built in command) and outputs the residual (number).