

The second test is in class on Friday 6 February, and covers sections 2.1–2.3 and 3.1–3.5.

Here are some sample questions, so that you have an idea of what to expect. The homework problems are also a good source of practice material.

1. Let  $a_n = \frac{n^2 + 3}{4n^2}$

- (a) Find  $a$  so that  $\lim_{n \rightarrow \infty} a_n = a$ .
- (b) Find  $N$  so that  $|a_n - a| < \epsilon$  whenever  $n > N$ .
- (c) Compute  $\lim_{n \rightarrow \infty} n(a_n - 1/4)$

2. You are reading a biology paper that gives the population model

$$N_{t+1} = \frac{25N_t}{13 + 3N_t^2},$$

where  $N_t$  represents the number of rabbits (measured in thousands) at time  $t$  (measured in years).

- (a) If this model is accurate, how many rabbits will there be in 10,000,000 years?
  - (b) What is the carrying capacity of this environment? Show that if the current population is greater than the carrying capacity then the population will decrease.
  - (c) If there are very few rabbits compared to the carrying capacity, then the population should grow exponentially, and the model should behave like  $N_{t+1} \approx RN_t$ . Determine the effective value of  $R$  used in this rabbit model. Explain your reasoning.
3. Compute the following limits. If you use the sandwich theorem, then indicate the two functions that you are using to sandwich.

(a)  $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 5x + 6}$

(b)  $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$

(c)  $\lim_{x \rightarrow -\infty} \frac{3e^{3x} - 4}{2e^{3x} - 2}$

(d)  $\lim_{x \rightarrow 0} x^2 \cos(3/x)$

4. Consider the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\ Ax & \text{if } x > -2 \end{cases},$$

where  $A$  is some constant.

- (a) Find  $\lim_{x \rightarrow -2^-} f(x)$ . Is  $f$  continuous from the left at  $x = -2$ ?
  - (b) What value of  $A$  would make  $f$  continuous at  $x = -2$ ?
  - (c) Using the value of  $A$  that you just found, graph  $f$ .
5. Use the Intermediate Value Theorem to show that the equation  $x^2 = \cos(x)$  has a solution.