Math 444/544 **Guide for Test 3**

Test 3 is on Tuesday 28 October, and covers Chapter 3. Here are some sample questions from Chapter 3.

1. If $x_0, x_1, \ldots, x_n$ are distinct numbers, then the Lagrange interpolating polynomial is given by
$$P(x) = \sum_{k=0}^{n} f(x_k) L_{n,k}(x).$$
Give the formula for $L_{n,k}(x)$ and then show that $P(x_j) = f(x_j)$ for all $j = 0, \ldots, n$.

2. We are given the following table of values for the function $f(x)$:

x_j	**
$f(x_j)$	**

Using the method of Newton's divided differences, construct the polynomial that interpolates this data.

3. If $x_0, x_1, \ldots, x_n$ are distinct numbers, then the Hermite interpolating polynomial is given by
$$H(x) = \sum_{k=0}^{n} f(x_k) H_{n,k}(x) + \sum_{k=0}^{n} f'(x_k) \hat{H}_{n,k}(x).$$
State the properties of $H_{n,k}(x)$ and $\hat{H}_{n,k}(x)$ and then use these to show that $H(x_j) = f(x_j)$ and $H'(x_j) = f'(x_j)$ for all $j = 0, \ldots, n$. What error bound do we have for Hermite interpolation?

4. We are given the following table of values and derivatives for the function $f(x)$:

x_j	**
$f(x_j)$	**
$f'(x_j)$	**

Construct the polynomial that interpolates this data. (I suggest using the method of Newton's divided differences.)

5. If $x_0, x_1, \ldots, x_n$ are distinct numbers, then there is a unique natural cubic spline that interpolates f at thes points. State the properties of this spline. What are the advantages and disadvantages of the spline interpolant versus a polynomial interpolant?

6. A Bézier curve is defined by left endpoint **, left guidepoint **, right endpoint **, and right guidepoint **. Sketch this curve.

7. **Math 444 students:** Make sure you wrote your name on the test.

   Math 544 students: The book has the following theorem:

   Theorem: Suppose $x_0, x_1, \ldots, x_n$ are distinct numbers in the interval $[a, b]$ and $f \in C^{n+1}[a, b]$. Then, for each x in $[a, b]$, a number $\xi(x)$ (generally unknown) in (a, b) exists with
$$f(x) = P(x) + \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x - x_0)(x - x_1) \cdots (x - x_n),$$
   where $P(x)$ is the Lagrange interpolating polynomial.

   Prove this theorem. If you use any other theorems from the book during your proof, then you need to state those theorems.