Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Your friend has a data set represented by vectors \( x \) and \( y \) and is considering using a polynomial interpolation, a spline interpolation, or a least squares approximation.

   (a) Explain to them how you get each of these in MATLAB.
   (b) Explain to them what each of these methods are.
   (c) Explain to them the advantages and disadvantages of each method. In particular, give an example situation when polynomial interpolation would be the best method, an example when splines would be the best method, and an example when least squares approximation would be best.

2. Estimate the integral \( \int_{0}^{4} x^2 \, dx \) using \( L_4, R_4, T_4 \) and \( S_4 \). Calculate the exact value and the errors of each of the approximations.

3. Write a MATLAB function program to do the Trapezoid Rule for integration of a function given by data. Let the inputs be vectors \( x \) and \( y \), representing a list of points \( (x_i, y_i) = (x_i, f(x_i)) \). Assume \( x_i < x_{i+1} \) but do not assume the \( x \) values are evenly spaced. Include comments.

4. Write a MATLAB function program to do the center-point method for integration of a function \( f(x, y) \) on a rectangle \( a \leq x \leq b, c \leq y \leq d \) using \( m \) subintervals in \( x \) and \( n \) subintervals in \( y \). Let the inputs be \( (f, a, b, c, d, m, n) \). Include comments.

5. Explain what would happen if you ran the following MATLAB commands:
   > format long
   > for i=1:30
   >     x=10^i+pi
   >     mypi=x-10^i
   >     error=(pi-mypi)/pi
   > end

6. Approximate the integral \( \int_{0}^{\pi} \sin x \, dx \) using \( M_4 \) and \( S_4 \). Which do you expect to be more accurate?

7. Write a MATLAB program to do the midpoint method for integration. Let the inputs be the function \( f \), the endpoints \( a, b \) and the number of subintervals \( n \).

8. Write a MATLAB function program to do the trapezoid method for integration. Let the inputs be the function \( f \), the endpoints \( a, b \) and the number of subintervals \( n \).


10. Describe and give formulas for 2 methods to approximate double integrals based on triangles.