Math 163A Guide for Test 4

Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Sketch the graph of a single function that has all of the following properties:

   (a) Continuous and differentiable everywhere except at \( x = -3 \), where it has a vertical asymptote.

   (b) A horizontal asymptote at \( y = 1 \).

   (c) An \( x \)-intercept at \( x = -2 \).

   (d) A \( y \)-intercept at \( y = 4 \).

   (e) \( f'(x) > 0 \) on the intervals \(( -\infty, -3 )\) and \(( -3, 2 )\).

   (f) \( f'(x) < 0 \) on the interval \(( 2, \infty )\).

   (g) \( f''(x) > 0 \) on the intervals \(( -\infty, -3 )\) and \(( 4, \infty )\).

   (h) \( f''(x) < 0 \) on the interval \(( -3, 4 )\).

   (i) \( f'(2) = 0 \).

   (j) An inflection point at \(( 4, 3 )\).

2. Let \( f(x) = \frac{x^3}{3} - 2x^2 + 3x + 1 \).

   (a) Find the domain of \( f \), any discontinuities, any asymptotes, and its \( y \)-intercept.

   (b) Find the critical points, extrema, and intervals of increase or decrease.

   (c) Find the intervals where it is concave up, and any inflection points.

   (d) Graph \( f(x) \), labeling the points that you found above.

3. Analyze and graph the function \( f(x) = x + \frac{9}{x} \).