Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Give MATLAB commands to plot the function \( f(x) = x^2 - 5 \) on the interval \([0, 8]\).

2. The function \( f(x) = x^2 - 5 \) is continuous and \( f(0) < 0 < f(8) \), so it has a zero on the interval \([0, 8]\). Perform 3 iterations of the bisection method to narrow down this interval.

3. Write a MATLAB function program to do \( n \) steps of the bisection method for a function \( f \) with starting interval \([a, b]\). Let \( f, a, b \) and \( n \) be the inputs and the final \( x \) be the output. Include comments.

4. For \( f(x) = x^2 - 5 \), do 2 iterations of Newton’s method, starting with \( x_0 \) as your best estimate from problem 2.

5. Write a MATLAB function program to do Newton’s method for a function \( f \) until \(|f(x)| < tol\). Let \( f, f', x_0 \) and \( tol \) be the inputs and the final \( x \) be the output. Include comments.

6. (a) We can approximate \( \sqrt{2} \) by setting \( f(x) = x^2 - 2 \) and solving for \( f(x^*) = 0 \). A reasonable starting guess is \( x_0 = 1 \). Do two iterations of Newton’s method to get a better approximation for \( \sqrt{2} \).
   
   (b) Write a MATLAB function with inputs \( z \) and \( tol \) that computes \( \sqrt{z} \) to tolerance \( tol \) using Newton’s method, and returns the result. Include comments. (It is illegal to use \texttt{sqrt} in your program.)

7. List your 10 least favorite MATLAB commands.

8. Write a MATLAB function program that calculates the sum of the squares of the first \( n \) integers.

9. For \( f(x) = x^2 - 5 \), do 2 iterations of Newton’s method, starting with \( x_0 = 2.0 \). What is the relative error of \( x_2 \)? About how many more steps would be needed to make the error less than \( 10^{-16} \)?

10. Write a MATLAB program to do \( n \) steps of Newton’s method for a function \( f \) with starting interval \([a, b]\). Let \( f, f', x_0 \) and \( n \) be the inputs and the final \( x \) the output.

11. For \( f(x) = x^2 - 5 \), do 2 iterations of the bisection method, starting with \([a, b] = [2, 3]\). What is the relative error? About how many more steps would be needed to make the error less than \( 10^{-6} \)?

12. Write a function program which will find the roots of a function \( f \) on an interval \([a, b]\).