

Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. (a) Using pivoting if needed, find the LU decomposition of

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 4 \end{bmatrix}.$$

- (b) Using your LU decomposition, solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = [3, 5]'$.
 - (c) Write a MATLAB program that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A and \mathbf{b} be the inputs and \mathbf{x} and the residual \mathbf{r} the outputs. Include comments.

2. What is the condition number of a matrix? How do you find it with MATLAB? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?

3. (a) Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

- (b) Describe how you could use the power method and inverse power method to find these two eigenvalues and eigenvectors.
4. Write a **Matlab** program to that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A , \mathbf{b} and tol be the inputs and \mathbf{x} the output. If the error (residual) is not less than tol , then display a warning.
5. Suppose $A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$. Using $\mathbf{v}_0 = (1, 1)'$ as the starting vector do 2 iterations of the Inverse Power Method for A . What do the results mean?
6. Suppose $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$. Using $\mathbf{v}_0 = (1, 1)'$ as the starting vector do 2 iterations of the Power Method for A . What do the results mean?
7. Write a **Matlab** program to do n iterations of the Power Method. Let the matrix A and n be inputs and let $[e \ \mathbf{v}]$ (the eigenvalue and eigenvector) be the outputs.
8. Give the MATLAB commands, or sequences of commands for solving a linear system $A\mathbf{x} = \mathbf{b}$ in as many ways as you know. Which of these are the worst and best?
9. What is the command in MATLAB to produce the eigenvalues and eigenvectors of a matrix. Which method does it use? What will be the form of the output?
10. Write a MATLAB function program that takes an input n , produces a random $n \times n$ matrix A and random vector $\bar{\mathbf{b}}$, solves $A\bar{\mathbf{x}} = \bar{\mathbf{b}}$ (using the built in command) and outputs the residual (number).
11. Write a MATLAB script program that will use Newton's method to find a root of the system of functions $f_1(x, y) = x^3 - y^2 + 1$ and $f_2(x, y) = y^3 + x - 1$ starting from the initial guess $(0, 0)$.