Math 163A  Guide for Test 3

Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Sketch the graph of a single function that has all of the following properties:
   (a) Continuous and differentiable everywhere except at $x = -3$, where it has a vertical asymptote.
   (b) A horizontal asymptote at $y = 1$.
   (c) An $x$-intercept at $x = -2$.
   (d) A $y$-intercept at $y = 4$.
   (e) $f'(x) > 0$ on the intervals $(-\infty, -3)$ and $(-3, 2)$.
   (f) $f'(x) < 0$ on the interval $(2, \infty)$.
   (g) $f''(x) > 0$ on the intervals $(-\infty, -3)$ and $(4, \infty)$.
   (h) $f''(x) < 0$ on the interval $(-3, 4)$.
   (i) $f'(2) = 0$.
   (j) An inflection point at $(4, 3)$.

2. Let $f(x) = \frac{x^3}{3} - 2x^2 + 3x + 1$.
   (a) Find the domain of $f$, any discontinuities, any asymptotes, and its $y$-intercept.
   (b) Find the critical points, extrema, and intervals of increase or decrease.
   (c) Find the intervals where it is concave up, and any inflection points.
   (d) Graph $f(x)$, labeling the points that you found above.

3. Analyze and graph the function $f(x) = x + \frac{9}{x}$.