Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Write a MATLAB program to do $n$ steps of the modified Euler method for a differential equation $\dot{x} = f(x, t)$, on the time interval $[a, b]$ with $x(a) = x_0$. Let the first line be:
   
   ```matlab
   function [t, x] = mymodeuler(f, x0, a, b, n).
   ```

2. Write a MATLAB program to do $n$ steps of the Euler method for a differential equation $\dot{x} = f(x, t)$, on the time interval $[a, b]$ with $x(a) = x_0$. Let the first line be:
   
   ```matlab
   function [t, x] = myeuler(f, x0, a, b, n).
   ```

3. Write the IVP: $\theta'' + 0.5\theta' + \sin \theta = \sin 2t$, $\theta(0) = 1$, $\theta'(0) = 0$ as a system of first order equations. Give all the MATLAB commands needed to solve this IVP on the interval $0 \leq t \leq 10$.

4. Describe RK45. What is the command for it in MATLAB?

5. What is variable step size? How is it implemented RK45?

6. Derive the explicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$, with boundary conditions, $u(0, t) = a$, $u(L, t) = b$, and $u(x, 0) = f(x)$.

7. Derive the implicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$.

8. When and why does the explicit finite difference method for the heat/diffusion equation become unstable?

9. Set up the finite difference equations for the BVP: $u_{xx} + u_{yy} = f(x, y)$, on the rectangle $0 \leq x \leq a$ and $0 \leq y \leq b$, with $u = 0$ on all the boundaries. Explain how the difference equations could be solved as a linear system.

10. Set up the finite difference equations for the BVP: $u_{rr} + \frac{1}{r}u_r = f(r)$, on the interval $0 \leq r \leq R$, with $u(R) = 4$ and $u_r(0) = 0$. Explain how to avoid the problem at $r = 0$.

11. Explain how to incorporate an insulated boundary in a finite difference method.

12. What is a finite element and a finite element solution?

13. What are main differences between the Finite Difference Method and Finite Elements Method?

14. If $U(x) = \sum_{j=1}^{n} C_j \Phi_j(\bar{x})$ is a finite element solution, what is the meaning of $C_j$? What is the final step of finding a finite element solution?

15. How are the boundary and interior values of the finite element solution obtained?