Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

You can use the following table of integrals for any of the questions:

\[ \int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln |x + \sqrt{x^2 + a^2}| + C \]

\[ \int \csc(x) \, dx = -\ln |\csc(x) + \cot(x)| + C \]

1. (a) \[ \int_0^3 \frac{1}{\sqrt{x}} \, dx = \]

(b) \[ \int_3^\infty \frac{1}{\sqrt{x}} \, dx = \]

(c) \[ \int_3^\infty \frac{1}{x^2} \, dx = \]

(d) \[ \int_3^\pi \frac{9}{\sqrt{x^2 - 4}} \, dx = \]

(e) \[ \int xe^{-3x} \, dx = \]

2. (a) \[ \int \frac{5}{x^2 + x - 2} \, dx = \]

(b) \[ \int 7x^2 \csc(x^3) \, dx = \]

(c) \[ \int \frac{x \ln(1 + x^2)}{1 + x^2} \, dx = \]

(d) \[ \int_1^4 (x - 2)^{-1/3} \, dx = \]

3. (a) Find the Taylor approximation of degree \( n = 3 \) about \( a = 1 \) for the function \( f(x) = \ln(x) \).

(b) Use your Taylor approximation to estimate the value of \( \ln(2) \).

(c) Use Taylor’s formula to bound the difference between your estimate and the true value \( \ln(2) \).