Math 266B Winter 2006 Guide for Final Exam

The final exam is Wednesday 15 March from 12:20-2:20pm.
Here are some sample questions, so that you have an idea of what to expect.

1. Suppose \( f \) is a differentiable function with the following properties:

\[
\begin{align*}
  f(0) &= e \\
  f(1) &= e^3 \\
  f(2) &= e^5 \\
  \int_0^1 f(x)dx &= \pi \\
  \int_1^2 f(x)dx &= \pi^3 \\
  \int_2^3 f(x)dx &= \pi^5
\end{align*}
\]

Evaluate the following. If one cannot be evaluated with the given information, write “Not Enough Information.” You do not need to justify your answer or show your work.

\[
\begin{align*}
  \int_0^1 f(r)dr &= , \int_0^2 f(x)dx = , \int_0^1 |f(x)|dx = , \int_0^3 f(x)dx = , \int_0^2 f'(x)dx = , \int_0^6 f(x)dx = \\
  \int_1^2 (f(x)+5)dx &= , \int_0^{14} f(x)dx - \int_2^{14} f(x)dx = , \int_1^3 f(1)dx = , \frac{d}{dt} \int_3^4 f(x)dx =
\end{align*}
\]

2. (a) \( \int x + \sin(7) + \frac{3}{1+x^2} - x^{-1/2} \, dx = \)

(b) \( \int_2^3 x \sin(5x) \, dx = \)

(c) \( \int_1^2 5x^2 e^{-x^3} \, dx = \)

(d) \( \int \frac{x^2}{2+x^3} \ln(2+x^3) \, dx = \)

3. (a) Solve the differential equation \( \frac{dr}{ds} = \frac{e^s}{\sin(r)} \), where \( s_0 = 5 \) for \( r_0 = 7 \).

(b) Solve the differential equation \( \frac{dN}{dt} = (N-1)(N-3) \) where \( N(0) = 7 \).

4. Dang! You are supposed to feed the tissue culture before leaving town for the weekend, but the bottle is empty and everyone else has left already. You normally give it 5\( ml \) of a solution that has 2\( g/ml \) glucose and 3\( g/ml \) sucrose. You find a blue bottle that has 30\( g/ml \) glucose and 20\( g/ml \) sucrose, and a green bottle that has 10\( g/ml \) glucose and 20\( g/ml \) sucrose. There is also a supply of distilled water. What do you do?

5. Solve the system of differential equations

\[
\begin{align*}
  \frac{dx_1}{dt} &= 4x_1 + 7x_2 \\
  \frac{dx_2}{dt} &= x_1 - 2x_2
\end{align*}
\]

with \( x_1(0) = 2 \) and \( x_2(0) = -1 \).